Mathematics 5710
Review : Test 2, Fall 2013

1. Given an event A, how do you find P(A) ?

Probability Rules the World

2. What is a random variable? Given an application that involves chance, how do you model or represent it mathematically?

3. What is a discrete random variable? What is the cumulative distribution function for a discrete random variable? What is the probability frequency function of a discrete random variable? How are they related? Do they characterize the random variable?

4. What is a continuous random variable? What is the cumulative distribution function for a continuous random variable? What is the probability density function of a continuous random variable? How are they related? Do they characterize the random variable?

5. List the most commonly used discrete distributions and state the defining condition for each.

6. List the most commonly used continuous distributions and state the defining condition for each.

7. In what way can a binomial distribution be approximately a Poisson distribution?

8. In what way can a hypergeometric distribution be approximately a binomial distribution?

9. In what way can a binomial distribution be approximately a normal distribution?

10. What is meant by the *memoryless* property for an exponential distribution?

11. If X is a normal distribution with parameters μ_X and σ_X , how do you compute probabilities involving X ?

12. Why do normal distributions arise so frequently in applications?

13. How are the Poisson, Exponential, and Gamma distributions related?

14. How is the Gamma function a generalization of the factorial function?

15. Suppose you know the distribution for a random variable X. How do you find the distribution of the random variable $Y = g(X)$?

Review Problems:

1. Suppose X is a random variable giving the number of tosses necessary for a biased coin to turn up heads where, on any single toss, the coin has probability 3/4 for "heads" and 1/4 for "tails". Find the probability that X is even.

2. Randomly draw one number from Box A = [1, 2, 3, 4] and another number from Box B = [1, 2, 3, 4 , 5]. Let the random variable *X* equal the maximum of the two draws.

a) Find *p(x)*, the probability frequency function of *X*.

b) Find $F_X(3)$

3. The number of flaws (bad records) on a computer tape follows a Poisson distribution with, on average, one flaw per 1200 feet. Let X be the number of flaws in a 4800 foot roll. Find $P(X > 2)$.

4. A point is chosen at random on a line segment of length 5. The line segment is then cut at this point and divided into two segments. Find the probability that the ratio of the shorter segment to the longer segment is less ¾.

5. Let $f(x) = cx^2$ for $0 < x < 2$.

a) Find the value of c that makes f(x) a density function for a random variable X .

b) Find the cumulative distribution function for X .

c) Find $P(X^2 < 2)$

6. The length of life in hours of an electronic component has an exponential distribution with an average life span of 500 hours.

a) Find the probability that a component lasts at least 800 hours.

b) Suppose a component has been in operation for 300 hours. What is the probability that it will last another 800 hours?

7. Suppose X is a normal random variable with $\mu = 5$ *and* $\sigma = 3$.

a) Express X in terms of Z (Z is standard normal, $\mu = 0$ *and* $\sigma = 1$)

b) Find $P(-1 < 2X - 3 < 1)$. Write your answer as an integral; do not evaluate.

8. Mathematics scores for USU students on the ACT test are normally distributed with $\mu = 20$ *and* $\sigma = 4$.

a) Find the probability that a randomly selected student has an ACT math score < 30.

b) What ACT score represents the 75-th percentile?

9. If the radius of a circle is an exponential random variable, find the density function for the area of the circle.

10. Suppose X is a standard normal random variable. Let $Y = X^2$.
Find the density function for Y and show that it is a Gamma density.