Math 5710 **Name** ____________________________

Final Test (Old 3)

1. Suppose events A , B and C are independent with respective probabilities $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{5}{6}$. Find the probability that exactly two of the events occur.

2. Suppose that a hand of 8 cards is dealt from a well-shuffled deck of cards. What is the probability of a *triple-triple-double***:** 3 cards of one face value, 3 cards of another face value, and 2 more cards of still another face value. (example: 3 sixes, 3 queens, and 2 aces)

3. A bin of 300 electrical components is known to contain 5 that are defective. Suppose the components are to be tested, one by one (randomly without replacement) until the defectives are discovered.

Find the probability of identifying the third defective on the 10^{th} draw.

4. Suppose that a cancer diagnostic test is 99 percent accurate both on those that do and those that do not have the disease. If 0.2 percent of the population have cancer, compute the probability that a tested person has cancer, given that his or her test result indicates so.

5. Let box A = [1, 2, 3] and box B = [1, 2, 3, 4]. Draw one number from each of the following boxes and let X be the maximum of the two numbers drawn.

a) Find the probability frequency function of X .

b) Find the moment generating function for X and use it to find the expected value of X.

6. Suppose that the number of flaws (bad records) on a computer tape follows a Poisson distribution with, on average, one flaw per 1600 feet. Let X be the number of flaws in a 4800 foot roll. Find the probability that $X > 2$.

7. Suppose E is an event that is described by two random variables X and Y. How do you find the probability of E?

8. Suppose X is an exponential random variable with $\lambda = 3$. Find the density function of the random variable $Y = X^4$.

9. The joint probability density function of random variables X and Y is given by

$$f(x,y) = \begin{cases} k\,x^2 y & \text{if } 0 < y \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the value of the constant k.

b) Find the density function for X, and the density function for Y.

c) Are X and Y independent?

d) Find the expected value of the random variable $W = \dfrac{Y}{X}$.

e) Find the conditional density of X given $Y = y$.

f) Find $P(X < \frac{1}{2}$ given $Y = y)$. Write your answer as an integral; do not evaluate.

10. Suppose X, Y are independent standard normal random variables. Find

$P(X^2 + Y^2 \leq 9)$. Write your answer as an integral; do not evaluate.

11. Suppose X, Y are random variables. Define what is meant by the covariance of X and Y (denoted by $\operatorname{cov}(X, Y)$).

12. a) State the Central Limit Theorem .

b) Suppose that $X_1, X_2, \ldots, X_n$ are repeated, independent measurements of a quantity μ and for each i, $E(X_i) = \mu$, $Var\, X_i = 25$. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. How large should n be so that $P\left(\left| \bar{X} - \mu \right| < 0.10 \right) \geq .95$?