Math 5710 **Name______________________**

Final Test (Old #2)

1. Events E, F, and G are independent and have respective probabilities $\frac{2}{5}$, $\frac{1}{4}$, and $\frac{2}{3}$. What is the probability that

a) Exactly two of the events occur?

b) At least one of them does <u>not</u> occur?

2. Suppose that a hand of 7 cards is dealt (without replacement) from a well-shuffled standard 52-card deck. The order in which you hold your cards does not matter. What is the probability of a triple and two pairs: 3 cards of the same face value, 2 more cards of another face value, and 2 more cards of still another face value?

3. A bin of 100 electrical components is known to contain 3 that are defective. Suppose the components are to be tested, one by one (randomly without replacement) until the defectives are discovered.

Find the probability of identifying the third defective on the 10^{th} draw.

4. Suppose X is a random variable giving the number of tosses necessary for a biased quarter to turn up heads where, on any single toss, the coin has probability $3/5$ for "heads" and $2/5$ for "tails". Find the probability that X is greater than 7.

5. Let box A = [1, 2, 3, 4] and box B = [1, 2, 3, 4, 5]. Draw one number from each of the following boxes and let X be the minimum of the two numbers drawn.

a) Find the probability frequency function of X .

b) Find the moment generating function for X and use it to find the expected value of X.

6. When a certain component of a manufacturing process breaks down, the time that it takes to fix it (in hours) is a random variable with the density function

$$f(x) = \begin{cases} 2e^{-2x} & \text{if } 0 \le x < \infty \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the probability that, when this component breaks down, it takes at most4 hours to fix it.

b) Derive the moment generating function for X.

7. Suppose X is a standard normal random variable ($\mu = 0$, $\sigma = 1$) and let $Y = X^2$. Use the 2-step method to find the density function for Y.

8. Suppose you draw two tickets (without replacement) from a box containing 3 blue tickets and 2 white tickets [B , B , B , W , W] . Let X be the number of blue tickets drawn and Y the number of white tickets drawn. Find the joint probability frequency function of X and Y .

9. The joint probability density function of random variables X and Y is given by

$$f(x, y) = \begin{cases} 8xy & \text{if } x \geq 0,\ y \geq 0 ,\ x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the marginal probability density functions of X and Y . Are X and Y independent?

b) Find $P(Y > X)$

10. Suppose X , Y are independent standard normal random variables. Find

$P(X^2 + Y^2 \leq 9)$. Write your answer as an integral; do not evaluate.

11. Suppose X , Y are random variables. Define what is meant by the covariance of X and Y (denoted by $\operatorname{cov}(X, Y)$).

12. a) State the Central Limit Theorem .

b) Suppose that $X_1, X_2, \ldots, X_n$ are repeated, independent measurements of a quantity μ and for each i , $E(X_i) = \mu$, $Var\, X_i = 36$. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. How large should n be so that $P\left(\left| \bar{X} - \mu \right| < 0.25 \right) \geq .95$?