Math 5710 **Name ____________________________**
Final Test (Old #1)

1. Suppose events A, B, C *and* D are independent with respective probabilities $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{5}{6}$. Find the probability that exactly three of the events occur.

2. Suppose that a hand of 10 cards is dealt from a well-shuffled deck of cards. What is the probability of a *triple-triple-double-double*: 3 cards of one face value, 3 cards of another face value, and 2 cards of another face value, and 2 cards of yet another face value. (example: 3 sixes, 3 queens, 2 fives, and 2 aces)

3. A bag of money contains twenty \$1 dollar bills and five \$100 bills. If you draw ten bills, one at a time without replacement, what is the probability of getting the last \$100 bill on the 10-th draw?

4. Suppose that a particular cancer diagnostic test is 99 percent accurate on those individuals who actually have cancer and is 95 percent accurate on those who do not have the disease. If .05 percent of the population has this type of cancer, compute the probability that a tested person has cancer, given that his or her test result indicates so.

5. Let box A = [1, 2, 3, 4, 5] and box B = [2, 3, 4]. Draw one number from each of the following boxes and let X be the minimum of the two numbers drawn.

a) Find the probability frequency function of X .

b) Find the moment generating function for X and use it to find the expected value of X.

6. The number of times that a person contracts a cold in a given year is approximately a Poisson random variable. On average, a person should expect 2 colds per year. Find the probability of getting more than three colds during the semester (a four-month period).

7. Suppose E is an event that is described by two random variables X and Y. How do you find the probability of E?

8. Suppose X is a standard normal random variable ($\mu = 0$, $\sigma = 1$) and let $Y = \sqrt{|X|}$. Find the density function for Y.

9. The joint probability density function of random variables X and Y is given by

$$f(x,y) = \begin{cases} k(x^2+y^2) & \text{if } 0 \le y \le 1-x^2 \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the density function for X .

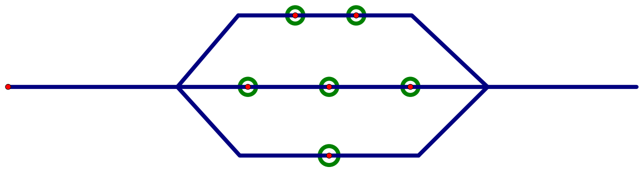
b) Find the density function for Y.

c) Are X and Y independent?

d) Find the conditional density of Y given $X = x$.

10. Suppose X *and* Y are independent random variables where X is standard normal and Y is exponential with parameter $\lambda = 5$. Find $P(\ X^2 + Y^2 \le\ 4\ )$. Write your answer as an integral; do not evaluate.

11. Each component of the following system has an independent exponentially distributed lifetime with parameter $\lambda = 7$. (The time until failure of each component is exponentially distributed with parameter 7.) Find the density function for the time until failure of the system.



12. Two hundred draws will be made at random with replacement from the box [0 , 0 , 1 , 1, 1] . For each *i*, let X_i be the number drawn on the *i*-th draw and let

$$\bar{X} = \frac{1}{200}\sum_{i=1}^{200} X_i \ .$$

Use the Central Limit Theorem to approximate $P(\bar{X} > 0.6346)$.