Math 4200 **Name____________________**
Final Test (Practice)

1. Show that $\sqrt{210}$ is an irrational number.

2. Suppose f is a function with domain I. Give the negation of the following statement:

For each a in I, $\forall \epsilon > 0, \exists\, \delta > 0$ such that if $|\,x - a\,| < \delta$ then $|\,f(x) - f(a)\,| < \epsilon$.

3. Let $S = \{(a, b) : a$ and b are integers and $ab \geq 0\}$. Show that S is a countable set.

4. Prove that there are uncountably many sequences of zeros and ones.

5. Prove: For each natural number n, $17^n - 13^n$ is divisible by 4.

6. Suppose $\{s_n\}$ is a monotone, non increasing sequence; that is, for each n, $s_n \geq s_{n+1}$. Suppose also that $\{s_n\}$ is bounded below; that is, there exists a real number M such that for all n, $s_n \geq M$. Prove that $\{s_n\}$ converges.

7. Suppose $\lim_{x \to b} r(x) = L$, $\lim_{x \to b} t(x) = L$, and for all x, $r(x) \leq s(x) \leq t(x)$.

Prove that $\lim_{x \to b} s(x) = L$.

8. Suppose h is continuous at $x = a$, and $\{c_n\}$ is a sequence such that $c_n \longrightarrow c$.
Show that $h(c_n) \longrightarrow h(c)$.

9. Suppose f is one-to-one, continuous on $[\,a, b\,]$, and $f(a) < f(b)$. Use the Intermediate Value Theorem to show that for all x in (a, b), $f(a) < f(x) < f(b)$.