Math 4200

Assignment 7

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1. Establish the existence or nonexistence of limits for the following sequences.

$a)$ $\left\{\frac{n}{n^2+1}\right\}$

$b)$ $\left\{1 + (-1)^n\right\}$

$c)$ $\left\{\frac{\sin n}{n}\right\}$

$d)$ $\left\{\sqrt{n+3} - \sqrt{n}\right\}$

$e)$ $\left\{\frac{2^n}{n!}\right\}$

$f)$ $\left\{\frac{1^2}{n^3} + \frac{2^2}{n^3} + \ldots + \frac{n^2}{n^3}\right\}$

2. Given $\epsilon = .0000005$, find a suitable M such that for all positive integers $n > M$,

$$\left|\frac{2n^3+5n}{3n^3-6} - \frac{2}{3}\right| < \epsilon \quad .$$

3. Prove that $\lim_{n\to\infty} a_n = L$ if and only if $\lim_{n\to\infty} b_n = 0$ where for each n in J,

$$b_n = a_n - L \quad .$$

4. If $\{ a_n \}$ and $\{ b_n \}$ are two convergent sequences satisfying $a_n \leq b_n$ for all n, can you conclude that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$?

5. Let $\{ a_n \}$ and $\{ b_n \}$ be sequences having limits A and B respectively. Prove that $\{ a_n + b_n \}$ converges to $A + B$.

6. Let $\{ a_n \}$ and $\{ b_n \}$ be sequences having limits A and B respectively. Prove that $\{ a_n \cdot b_n \}$ converges to $A \cdot B$.

7. Suppose $\{ a_n \}$ converges to 0, and $\{ b_n \}$ is bounded. Show that $\{ a_n \cdot b_n \}$ converges to 0 .