Math 4200, Assignment 5

Text Exercises: Induction - 19, 20, 21, 24

1. What is wrong with the proof given below that all horses have the same color?

Proof: Let n be the number of horses. When n = 1, the statement is clearly true, that is, one horse has the same color, whatever color it is. Assume that any group of n horses has the same color. Now consider a group of (n+1) horses. Taking any n of them, the induction hypothesis states that they all have the same color, say brown. The only issue is the color of the remaining "uncolored" horse. Consider, therefore any other group of n of the (n+1) horses that contains the uncolored horse. Again, by the induction hypothesis, all of the horses in the new group must have the same color. Then, since all of the colored horses in this group are brown, the uncolored horse must also be brown.

2. Prove that $1^3 + 2^3 + 3^3 + ... + n^3 = (1+2+3+...+n)^2$ for all positive integers n.

3. Prove that $11^n - 4^n$ is divisible by 7 for all positive integers n.

4. Conjecture and prove a closed formula for the product

$$(1-\frac{1}{4})\,(1-\frac{1}{9})\,...\,(1-\frac{1}{n^2})$$

5. For each $n \in J$, let $S(n)$ denote the assertion "$n^2 + 5n + 1$ is an even integer."

a) Show that $\forall n \in J$, $S(n)$ implies $S(n+1)$.
b) For what values of n is $S(n)$ true?
c) What is the point of this exercise?