

Math 4200
Assignment 3

1. Suppose that a hand of 7 cards is dealt from a well-shuffled deck of cards. How many ways can you be dealt a *triple-double-double*: 3 cards of one face value, 2 cards of another face value, and 2 more cards of still another face (for example, 3 sixes, 2 queens, and 2 aces)?

2. From a group of 8 women and 6 men a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if 1 man and 1 woman refuse to serve together?

3. Prove the following identity:

$$\sum_{k=0}^n \binom{n}{k} \binom{m-n}{n-k} = \binom{m}{n}$$

Hint: Consider a class with n girls and $m-n$ boys.

4. What is the coefficient of x^3 in the expansion of $\left(2x + \frac{1}{x}\right)^9$?

5. Ten points are chosen on or inside an equilateral triangle of side 1. Use the Pigeon Hole Principle to prove that at least two of the points are no more than $1/3$ unit apart.

6. Suppose n people are at a party. Use the Pigeon Hole Principle to show that at least two of the people know the same number of people among those present. Assume that if A knows B, then B knows A.