

MATHEMATICS 4200
Assignment 2.

1. Write each of the following statements equivalently in the form *if p then q* and then write the converse and contrapositive.

- a) Whenever the phone rings, I run to answer it.
- b) The earth will not rotate if $x^2 + y^2$ is negative.
- c) It is necessary for you to eat in order to live.
- d) A sufficient condition for getting good grades is to be a genius.

2. Construct a truth table for each of the following statements.

- a) $(p \Rightarrow q) \Rightarrow r$
- b) $(p \text{ and } \sim r) \Rightarrow \sim q$

3. Show that the following statements are logically equivalent.

- a) $p \Rightarrow q$
- b) $\sim (p \text{ and } \sim q)$
- c) $\sim q \Rightarrow \sim p$
- d) $q \text{ or } \sim p$

4. Give a logical negation of each of the following statements.

- a) Some problems are not easy.
- b) $\exists x$ such that $x > 0$ and $f(x) = 4$.
- c) $\exists x$ such that $f(x) = 0$ or $f(x) = x$.
- d) $\forall x$, there exists y such that $f(x,y) = 0$.

5. Rewrite the following statements using the symbols \forall and \exists , and then obtain a logical negation of each.

- a) Given y in A , there exists an x in the domain of f such that $y = f(x)$.
- b) If x, y are in the domain of f , then $x < y$ implies $f(x) < f(y)$
- c) Given $\epsilon > 0$, there exists a real number $N > 0$ such that if $n > N$, then $|a_n - L| < \epsilon$.
- d) Given $\epsilon > 0$, there exists a real number $\delta > 0$ such that for all x in the domain of f , $|x - c| < \delta$ implies $|f(x) - f(c)| < \epsilon$.
- e) Given $\epsilon > 0$, there exists a real number $\delta > 0$ such that for all x, y in A , $|f(x) - f(y)| < \epsilon$ whenever $|x - y| < \delta$.

6. Read the Wikipedia article on History of Logic. The link is on the *Math 4200* website, or go to <http://en.wikipedia.org/wiki/Logic>.