

**MATHEMATICS 4200**  
**Assignment 1.**

1. An integer  $n$  is said to be **even** provided  $n = 2k$  for some integer  $k$ ;  $n$  is **odd** provided  $n = 2j + 1$  for some integer  $j$ . Prove each of the following statements.

- a) The product of any two even integers is even.
  - b) The product of any two odd integers is odd.
  - c) If  $n^2$  is an even integer, then  $n$  is even.
  - d) If  $n^2$  is an odd integer, then  $n$  is odd.
  - e) The sum of any two odd integers is even.
2. Prove: If  $x^2 + x - 6 \geq 0$  then  $x \leq -3$  or  $x \geq 2$ .
3. Prove or give a counterexample:  $3^n + 2$  is prime for all positive integers  $n$ .
4. Prove or give a counterexample: there do not exist rational numbers  $x$  and  $y$  such that  $x^y$  is a positive integer and  $y^x$  is a negative integer.
5. Show that  $\sqrt{3}$  is irrational.
6. If  $p$  is a prime number, show that  $\sqrt{p}$  is irrational.
7. Show that if the positive integer  $n$  is not a perfect square, then  $\sqrt{n}$  is irrational.
8. Show that  $\log_2 3$  is irrational.
9. Show that the square root of an irrational number is irrational.
10. When were  $\pi$  and  $e$  proved to be irrational numbers?