Trigonometric Integrals

Example 1:

Evaluate $\int \sin^3 x\,dx$.

$= \int \sin^2 x \cdot \sin x \, dx$

$= \int (1-\cos^2 x)\sin x \, dx$

Let $u = \cos x$, $du = -\sin x \, dx$

$\int -(1-u^2)\,du = \int (u^2-1)\,du$

$= \frac{u^3}{3} - u + C$

$= \frac{\cos^3 x}{3} - \cos x + C$

Example 2:

Evaluate $\int \sin^3 x \cos^4 dx$.

$= \int (\sin^2 x) \cos^4 x \cdot \sin x \, dx$

$= \int (1 - \cos^2 x) \cos^4 x \sin x \, dx$

Let $u = \cos x$ $\quad du = -\sin x \, dx$

$\int -(1 - u^2) u^4 \, du = \int (u^6 - u^4) \, du$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

Example 3:

Evaluate $\int \sin^2 x \cos^2 dx$.

Use $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$= \int \frac{1-\cos^2 2x}{4}\, dx = \frac{1}{4}\int 1 - \left[\frac{1+\cos 4x}{2}\right] dx$

$= \frac{1}{4}\int \left(\frac{1}{2} - \frac{\cos 4x}{2}\right) dx$

$= \frac{1}{4}\left[\frac{x}{2} - \frac{\sin 4x}{8}\right] + C$

Example 4:

Evaluate $\int_0^{\pi} \sin^2 x \, dx$.

use $\sin^2 x = \frac{1-\cos 2x}{2}$.

$$= \int_0^{\pi} \frac{1-\cos 2x}{2} \, dx$$

$$= \left(\frac{x}{2} - \frac{\sin 2x}{4} \right) \Big|_0^{\pi}$$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$$