Integration by Parts

Recall the product rule:

$$\frac{d}{dx}[\ f(x)g(x)\ ] = f'(x)g(x) + f(x)g'(x)$$

$$Therefore \int [\ f'(x)g(x) + f(x)g'(x)\ ]\,dx = f(x)g(x) + c\ .$$

$$We\,now\,obtain \int f(x)g'(x)\ ]\,dx = f(x)g(x) - \int [\ g(x)f'(x)\ ]dx\ .$$

With $u = f(x)$ *and* $v = g(x)$ *we obtain*
the integration by parts formula.

$$\int u\,dv = uv - \int v\,du$$

Example 1:

Evaluate $\int x\sin x\,dx$.

Let $u = x$, $dv = \sin x\,dx$.

Then, $du = dx$, $v = -\cos x$.

$$= -x\cos x - \int -\cos x\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$

Example 2:

Evaluate $\int 6x^2 \ln x \, dx$.

Let $u = \ln x$, $dv = 6x^2 dx$.

Then $du = \frac{1}{x} dx$, $v = 2x^3$

$$= 2x^3 \ln x - \int 2x^3 \cdot \frac{1}{x} dx$$

$$= 2x^3 \ln x - \int 2x^2 dx$$

$$= 2x^3 \ln x - \frac{2x^3}{3} + C$$

Example 3:

Evaluate $\int_0^1 xe^x \, dx$.

Let $u = x$, $dv = e^x dx$.

So $du = dx$, $v = e^x$.

$$= xe^x \Big|_0^1 - \int_0^1 e^x dx$$

$$= e - e^x \Big|_0^1 = e - (e - 1) = 1.$$