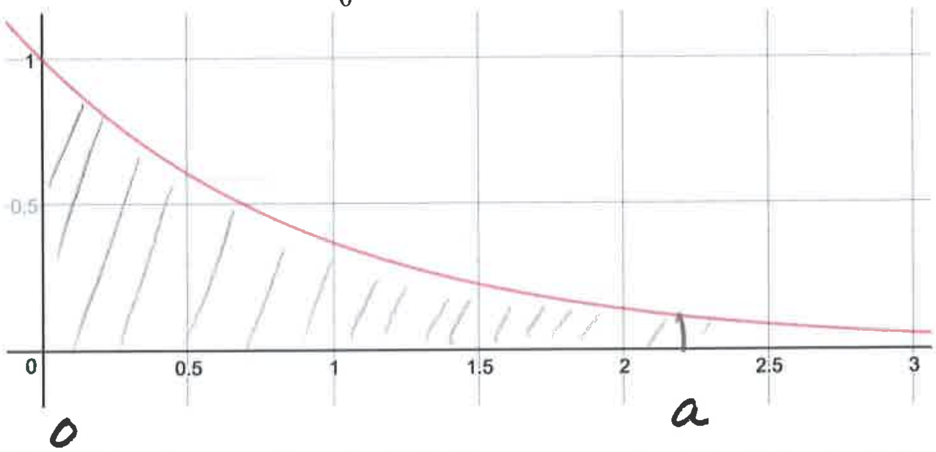
**Improper Integrals**

Example 1:

Determine whether $\int_0^{\infty} e^{-x}\,dx$ converges.



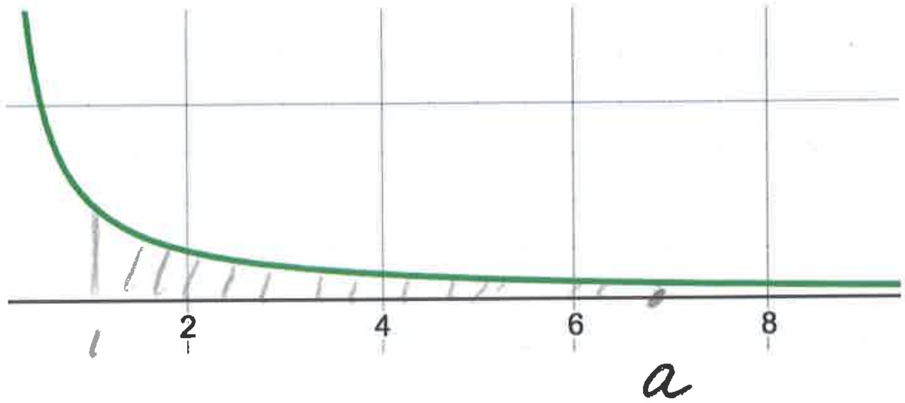

$$\lim_{a\to\infty} \int_0^a e^{-x}\,dx = \lim_{a\to\infty} -e^{-x}\Big|_0^a$$

$$= \lim_{a\to\infty} \left(-e^{-a}+1\right) = \lim_{a\to\infty} \left(-\frac{1}{e^a}+1\right)$$

$$= 1$$ (converges)

Example 2:

Determine whether $\int_1^{\infty} \frac{1}{x}\, dx$ converges.



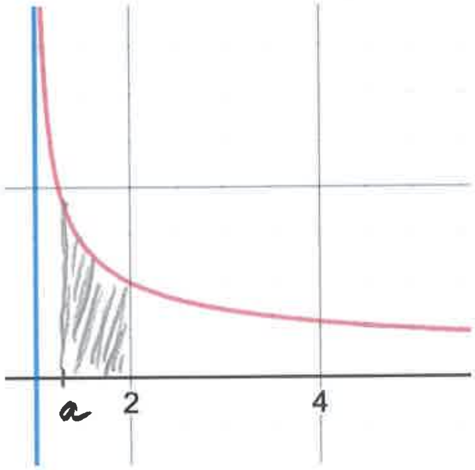

$$= \lim_{a\to\infty} \int_1^{a} \frac{1}{x}\, dx = \lim_{a\to\infty} (\ln x)\Big|_1^{a}$$

$$= \lim_{a\to\infty} (\ln a - 0) = \infty$$

(diverges)

Example 3:

Determine whether $\int_1^2 \frac{1}{\sqrt{x-1}}\,dx$ converges.




$$= \lim_{a \to 1^+} \int_a^2 \frac{1}{\sqrt{x-1}}\,dx$$

$$= \lim_{a \to 1^+} \left(2\sqrt{x-1}\right)\Big|_a^2$$

$$= \lim_{a \to 1^+} \left(2 - 2\sqrt{a-1}\right) = 2$$

(converges)

Example 2: