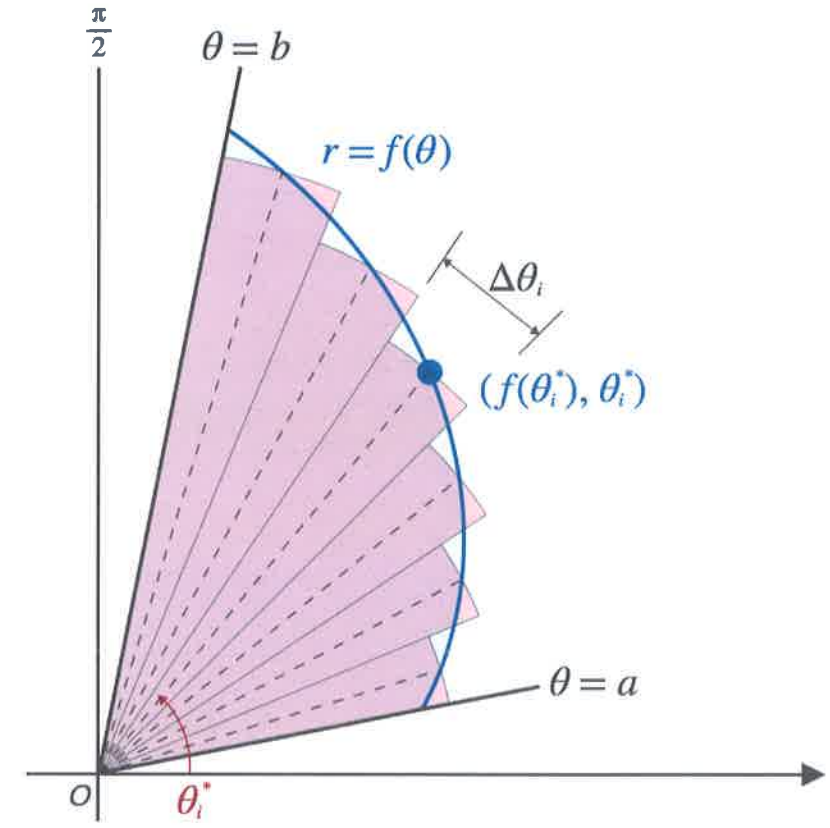
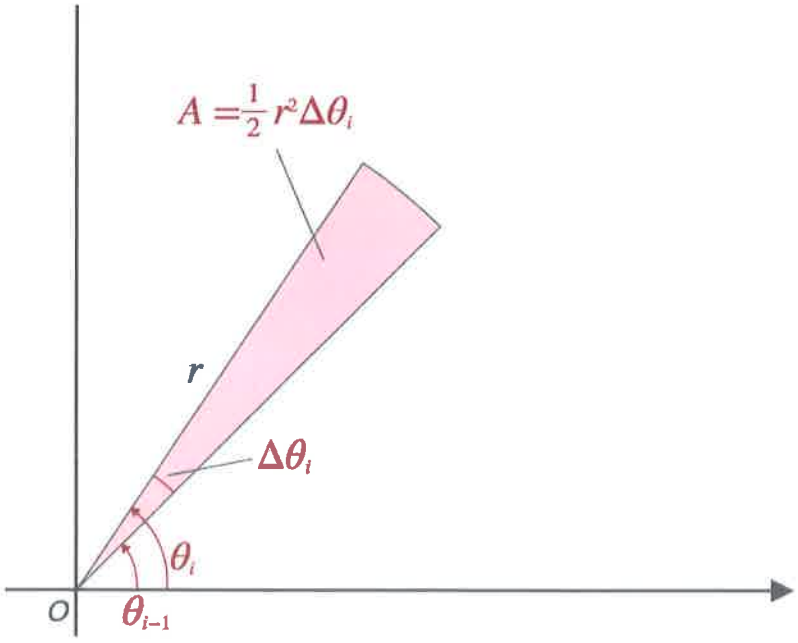
Area of a Polar Region:



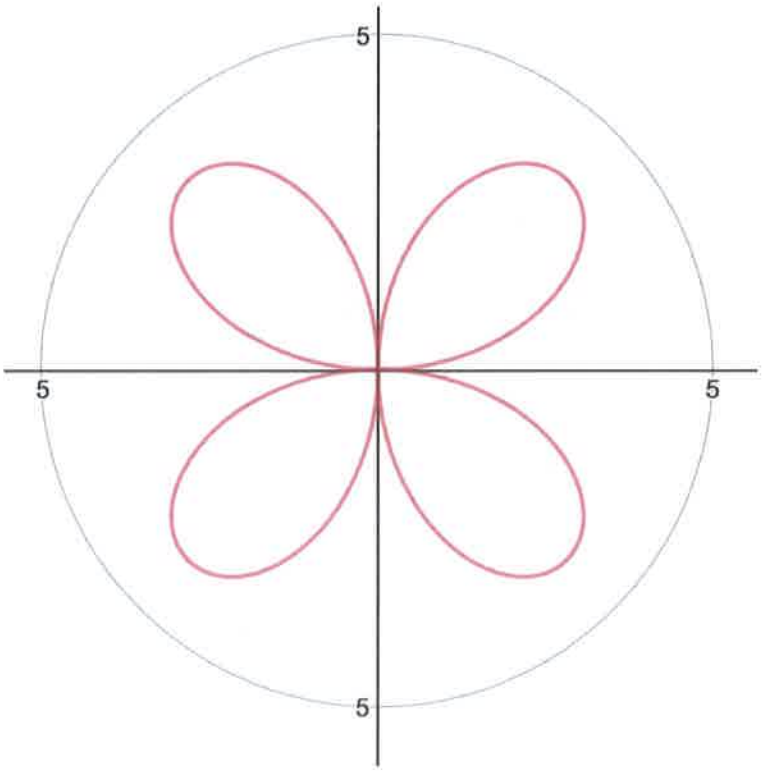


Formula: Area of a Polar Region

The area A of a region bounded between two rays $\theta = a$ and $\theta = b$, where $0 < b - a \leq 2\pi$, and by the polar equation $r = f(\theta)$, where f is a continuous positive function, is given by

$$A = \int dA = \int_a^b \frac{1}{2}[f(\theta)]^2 d\theta.$$

Find the area enclosed by the polar curve $r = 4sin2\theta$.



$$A = \int_a^b \frac{1}{2}[f(\theta)]^2 d\theta$$

$$= 4\int_0^{\frac{\pi}{2}} \frac{1}{2}[4\sin 2\theta]^2 d\theta$$

$$= 32\int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta$$

$$= 32\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 4\theta}{2}\right) d\theta$$

$$= 32\left[\frac{\theta}{2} - \frac{\sin 4\theta}{8}\right]_0^{\frac{\pi}{2}}$$

$$= 32\left[\frac{\pi}{4} - \frac{\sin 2\pi}{8}\right] - 0$$

$$= 32 \cdot \frac{\pi}{4} = 8\pi$$