Geometic Series

For $r \neq 1$, the sum of the first n terms of a geometric series is

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \sum_{k=0}^{n-1} ar^k = a\,\frac{1 - r^n}{1 - r},$$

$$\begin{aligned} s &= a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}, \\ rs &= ar + ar^2 + ar^3 + ar^4 + \cdots + ar^n, \\ s - rs &= a - ar^n, \\ s(1 - r) &= a(1 - r^n), \end{aligned}$$

$$s = a\frac{1 - r^n}{1 - r} \quad (\text{if } r \neq 1).$$

As n goes to infinity

$$a + ar + ar^2 + ar^3 + ar^4 + \cdots = \sum_{k=0}^{\infty} ar^k = \frac{a}{1 - r}, \text{ for } |r| < 1.$$

When $a = 1$

$$1 + r + r^2 + r^3 + \cdots = \frac{1}{1 - r},$$

The geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ converges if $|r| < 1$ and diverges if $|r| \geq 1$. When $|r| < 1$.

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1 - r}$$ First term divided by the difference between one and the common ratio.

Example 1.

Find all values of x for which the geometric series $\sum_{n=1}^{\infty} 5(4-5x)^{n-1}$ converges.

Example 2.

Express the repeating decimal $.4\overline{3429}$ as a geometric series and evaluate the series to express the decimal as a rational number.

Example 3.

A spring is attached to a platform so that it is hanging down vertically. A 1 pound weight is attached and released. The tension in the spring is such that it extends 12 inches and then rebounds up 9 inches. If the oscillations continue, what is the total distance the weight has traveled when the spring reaches equilibrium?