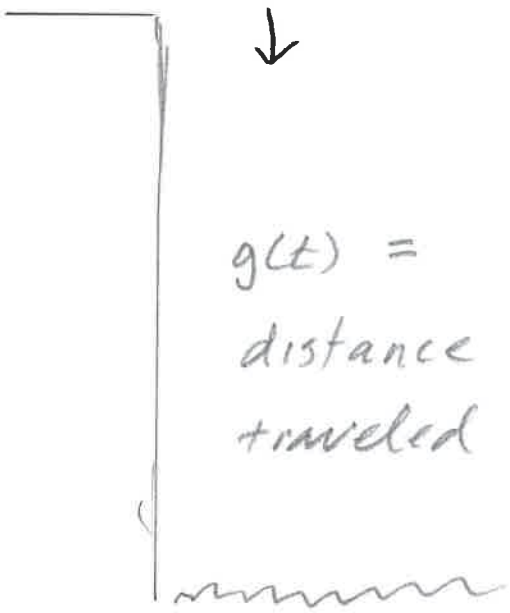
Introduction to Calculus, Example 1 (Newton)

You drop a rock from the top of a cliff at Lake Powell that is 220 feet above the water. How fast is the rock traveling when it hits the water?



Galileo: $g(t) = 16t^2$

$g(t_0)$ { t_0 } $g(t_0+h)$

t_0+h

$$\frac{g(t_0+h)-g(t_0)}{h} = \text{average velocity over } [t_0, t_0+h]$$

$$= \frac{16(t_0+h)^2 - 16t_0^2}{h}$$

$$= \frac{16(t_0^2+2ht_0+h^2) - 16t_0^2}{h}$$

$$= \frac{32ht_0+h^2}{h} = \frac{h}{h}(32t_0+h)$$

$= 32t_0+h$ when $h \neq 0$. What happens to this average velocity when h gets smaller and smaller? We get the velocity or speed at time t_0, $32t_0$.

Lake Powell: How long does it take for the rock to hit the water?

$g(t) = 16t^2 = 220, \quad t^2 = 3.7$

velocity $= 32(3.7) \frac{ft}{sec}$

$= 118.4 \frac{ft}{sec}$

$118 \frac{ft}{sec} \cdot 60 \frac{sec}{min} \cdot 60 \frac{min}{hr} \cdot \frac{1 \text{ mile}}{5280 ft}$

$\approx$ 80 mile per hour!

(Don't jump!)