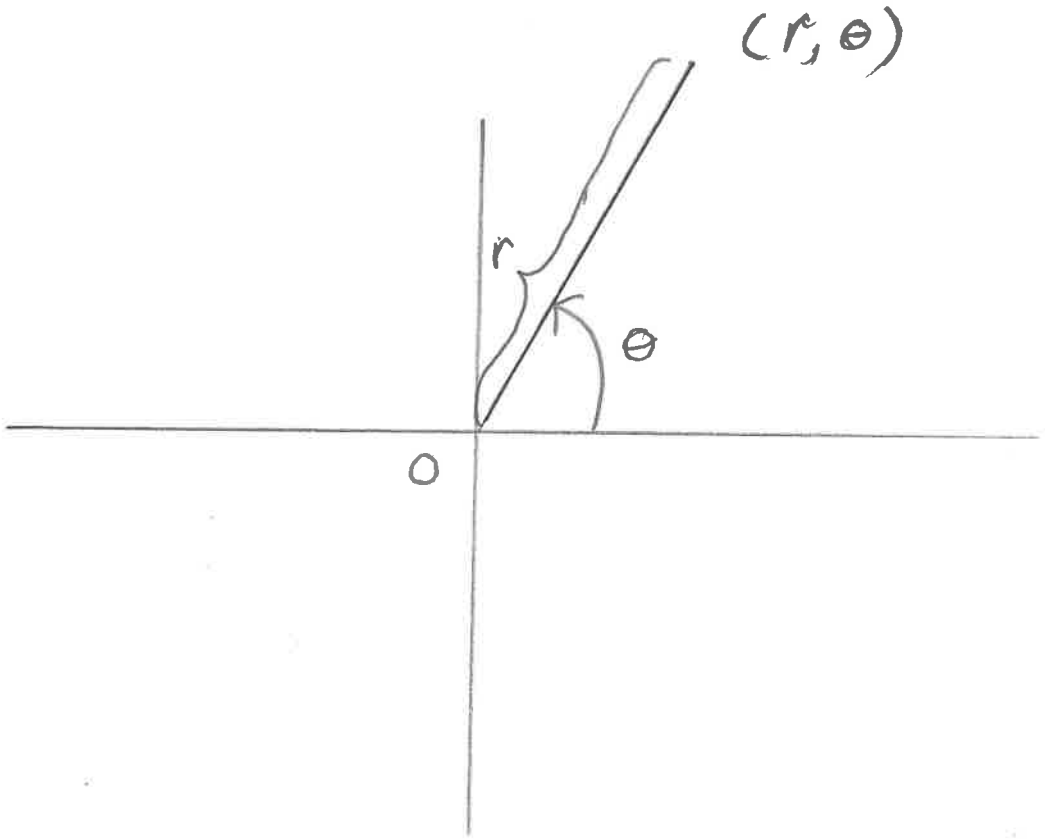
Polar Coordinates

Representing Points Using Polar Coordinates (r, θ):

To define polar coordinates in a plane, we first an origin O (called the **pole**) and an initial ray from O.

·θ represents a directed angle rotated from the initial ray.

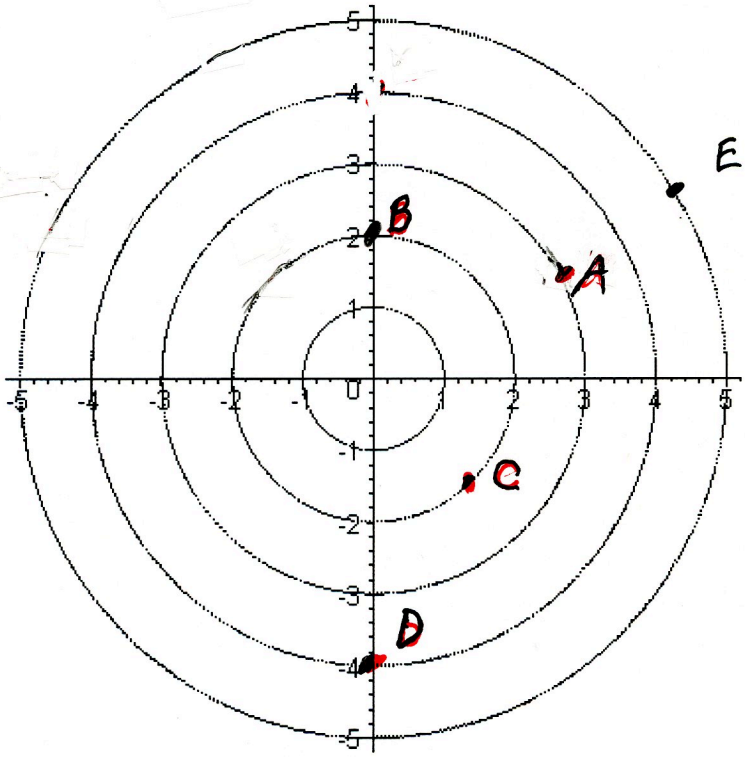
·r presents a directed distance from O.



Plot the following points whose polar coordinates are given:

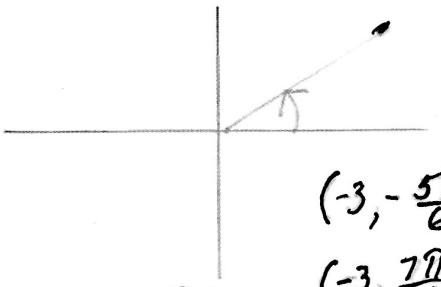
$A(3, \frac{\pi}{6})$ $B(2, 90^o)$ $C(-2, \frac{3\pi}{4})$

$D(4, -\frac{\pi}{2})$ $E(-5, -150^o)$



Rewrite each of the following points using four different sets of polar coordinates.

$A(3, \frac{\pi}{6})$



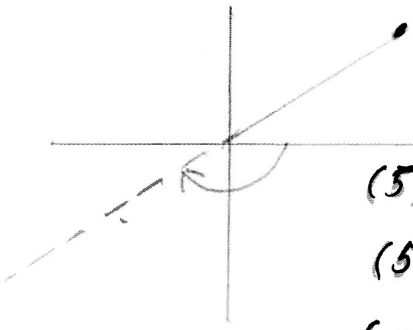
$(3, \frac{13\pi}{6})$

$(-3, -\frac{5\pi}{6})$

$(-3, \frac{7\pi}{6})$

$(-3, -\frac{5\pi}{6} - 2\pi)$

$B(-5, -\frac{5\pi}{6})$



$(5, \frac{\pi}{6})$

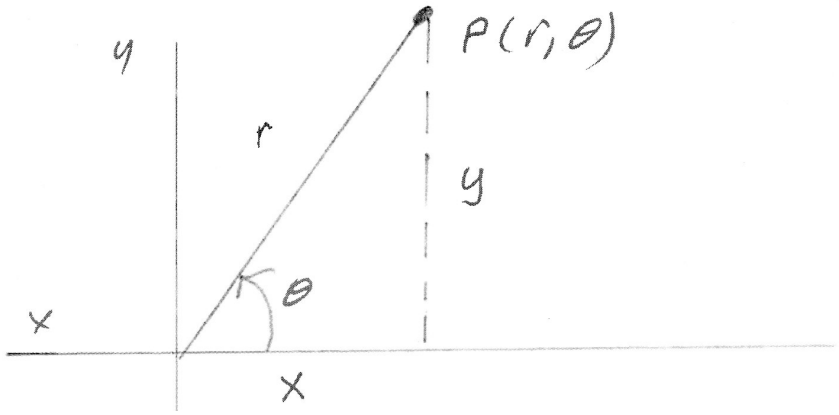
$(5, \frac{\pi}{6} + 2\pi)$

$(5, -\frac{11\pi}{6})$

$(-5, -\frac{5\pi}{6} - 2\pi)$

Conversion Equations for Polar and Rectangular Coordinates:

$x = r\cos\theta$ $\quad$ $y = r\sin\theta$ $\quad$ $r^2 = x^2 + y^2$

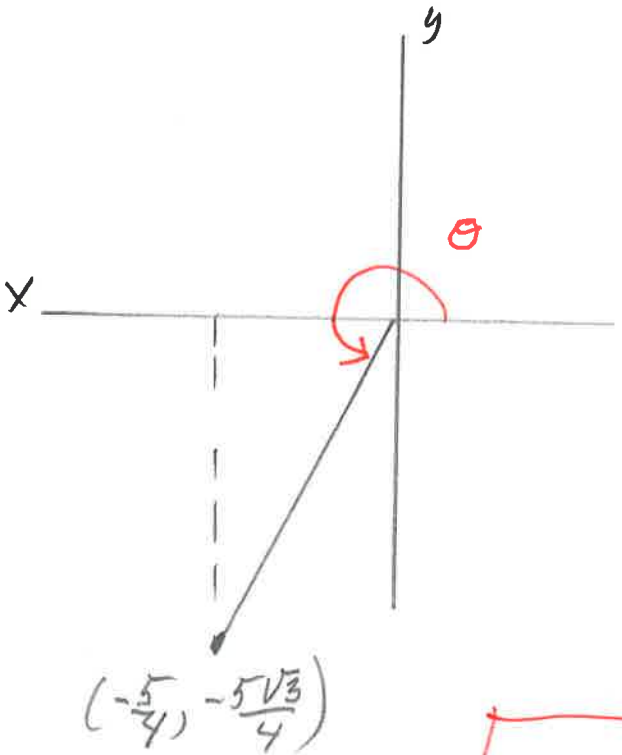


Convert the point whose polar coordinates are $(5, \frac{3\pi}{4})$ into rectangular coordinates.

$$x = 5\cos\frac{3\pi}{4} = -\frac{5\sqrt{2}}{2}$$

$$y = 5\sin\frac{3\pi}{4} = \frac{5\sqrt{2}}{2}$$

Convert the point whose rectangular coordinates are $(-\frac{5}{4}, -\frac{5\sqrt{3}}{4})$ into polar coordinates.



Take θ in the third quadrant.

$$r^2 = x^2 + y^2 = \frac{25}{16} + \frac{75}{16}$$

$$r^2 = \frac{100}{16}, \quad r = \frac{10}{4}$$

$$\sin\theta = \frac{y}{r} = \frac{-5\sqrt{3}}{4} \Big/ \frac{10}{4}$$

$$\sin\theta = -\frac{\sqrt{3}}{2}, \quad \pi + \frac{\pi}{3}$$

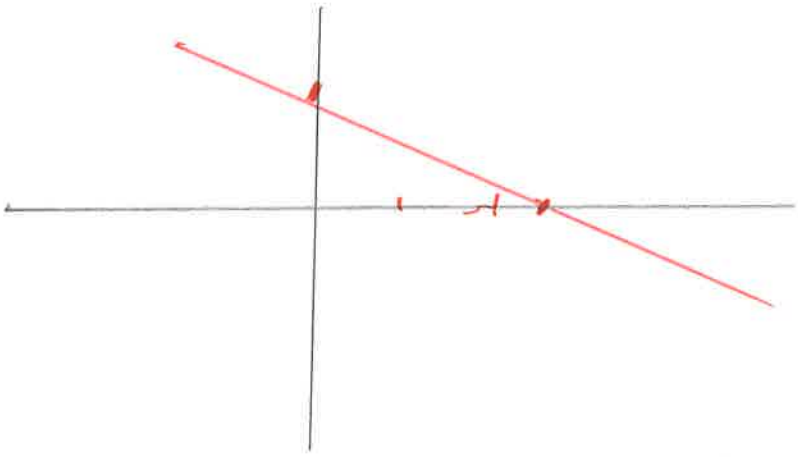
$$\theta = \frac{4\pi}{3}$$

$$\left(\frac{5}{2}, \frac{4\pi}{3}\right)$$

Convert the equation $2x + 3y = 5$ into a polar equation and then graph the equation on your calculator.

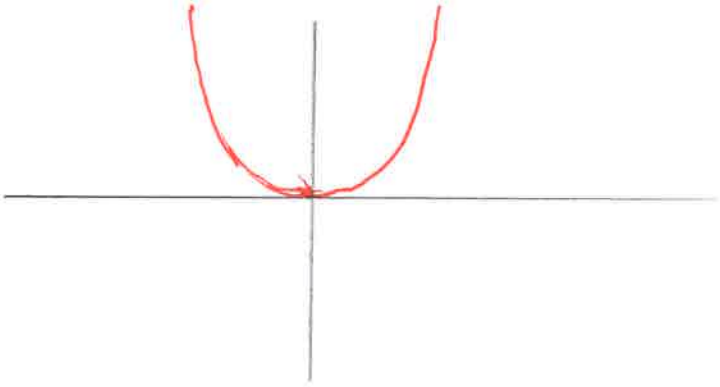
$2r\cos\theta + 3r\sin\theta = 5$

$r[2\cos\theta + 3\sin\theta] = 5$



Convert the equation $y = x^2$ into a polar equation and then graph the equation on your calculator.

$r\sin\theta = r^2\cos^2\theta$,



Convert the equation $r = 2\cos\theta$ into a rectangular equation.

$r^2 = 2r\cos\theta$

$x^2 + y^2 = 2x$

Convert the equation $r = \cos\theta + 2\sin\theta$ into a rectangular equation.

$r^2 = r\cos\theta + 2r\sin\theta$

$x^2 + y^2 = x + 2y$

Polar Equations and Their Graphs

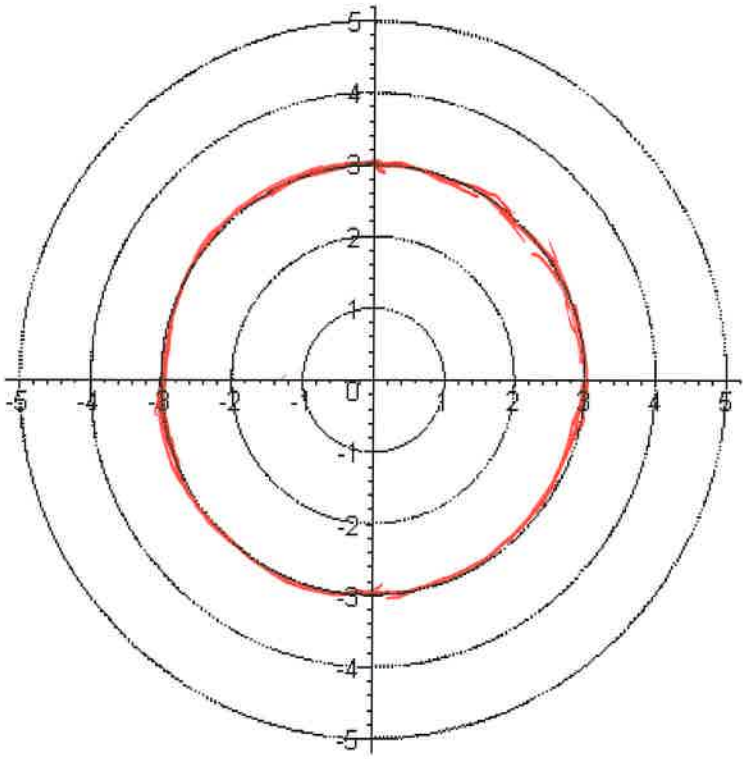
What is the graph of an equation?

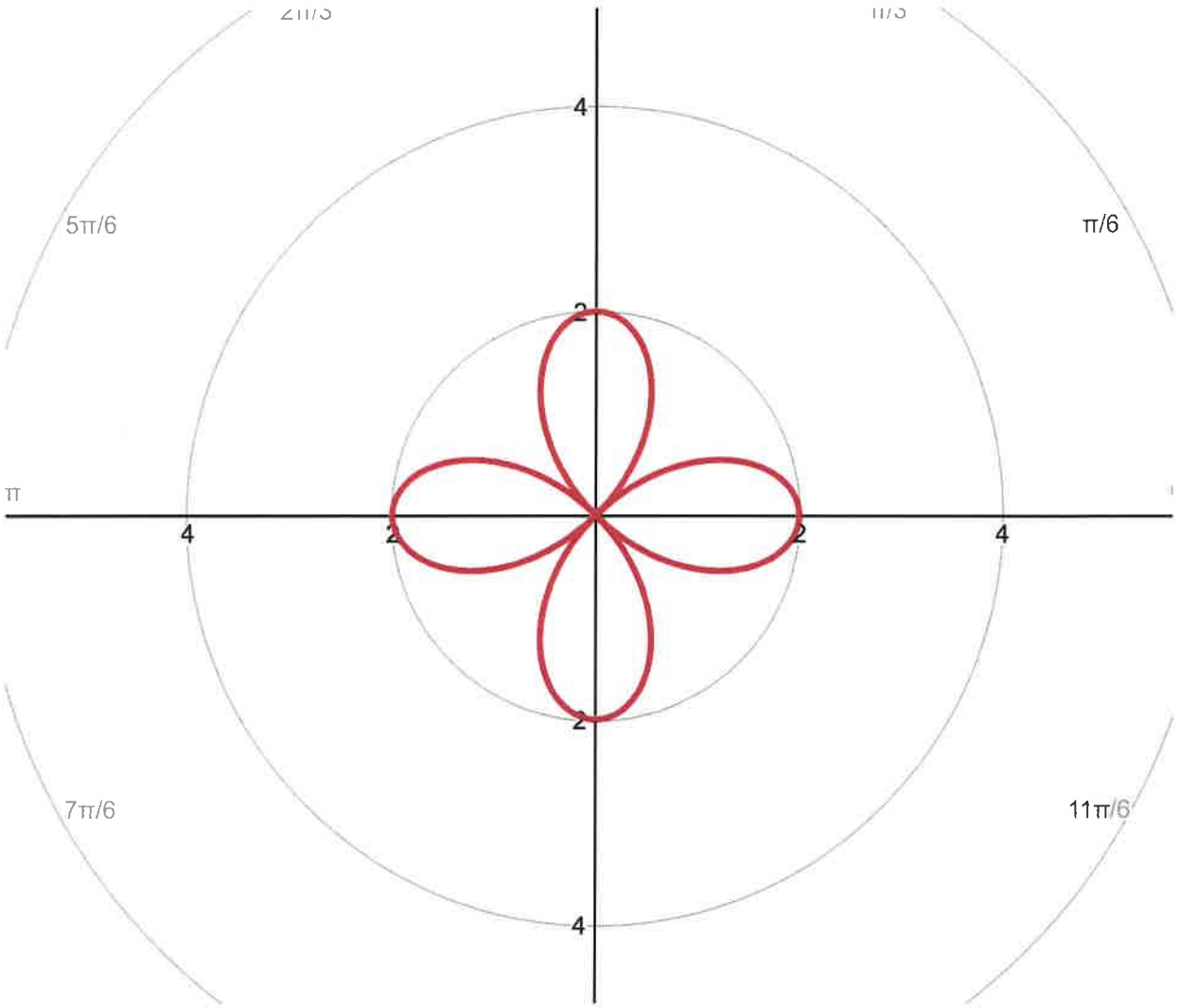
$\{(r, \theta)$ which satisfy the equation$\}$

What does the graph of the polar equation $r = 3$ look like?

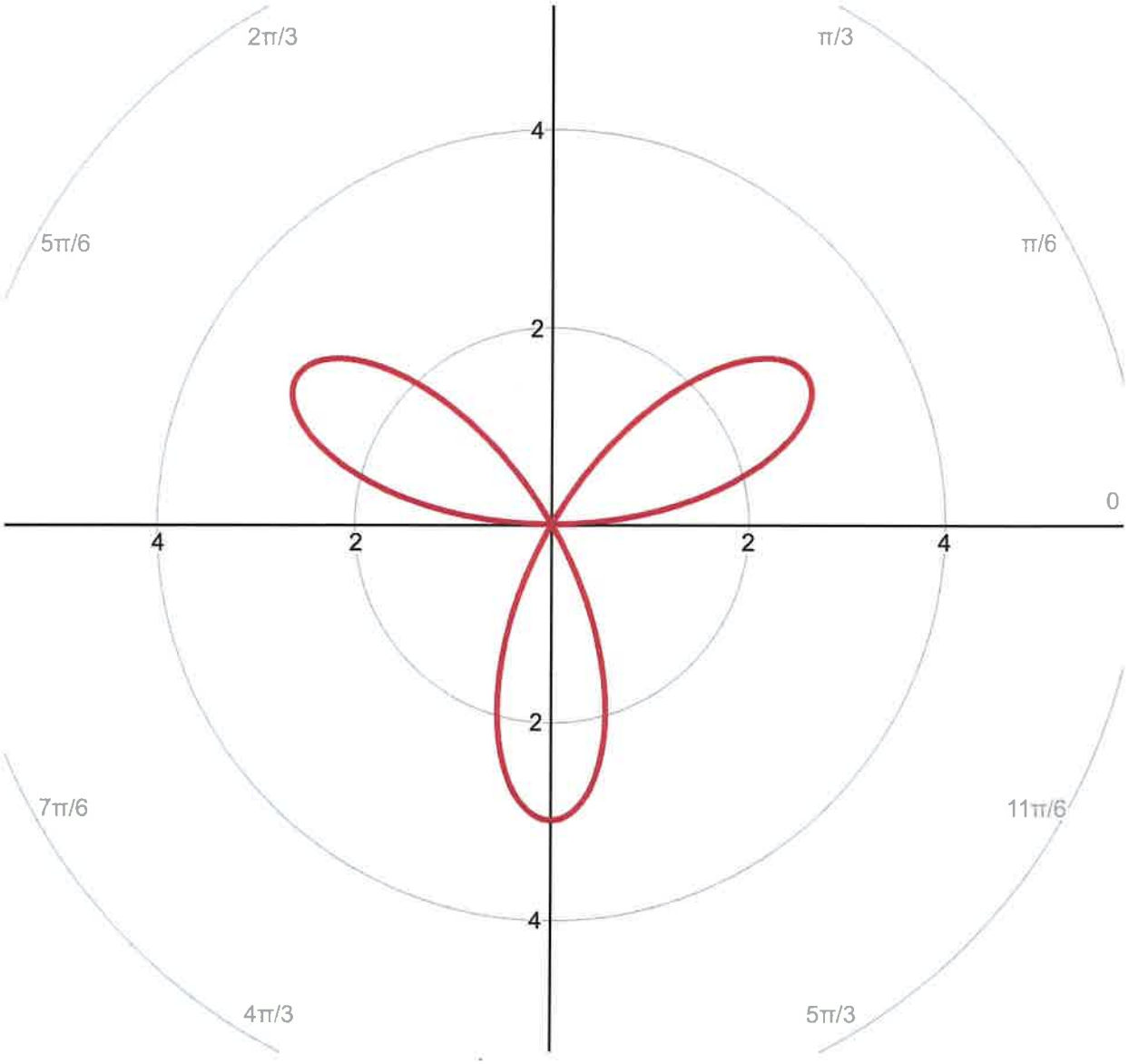
θ can be any value

circle with radius 3.





$r = 2\cos(2\theta)$



$r = 3 \sin(3\theta)$