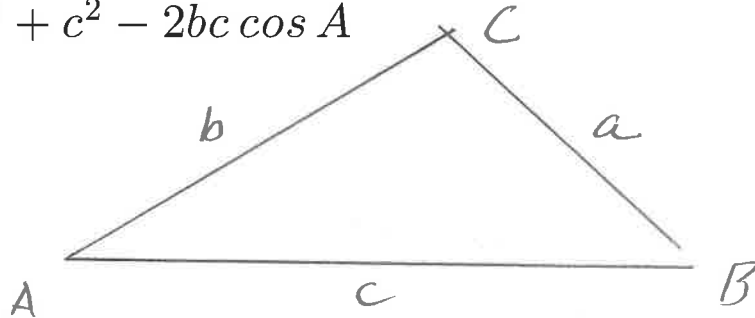


The Law of Cosines

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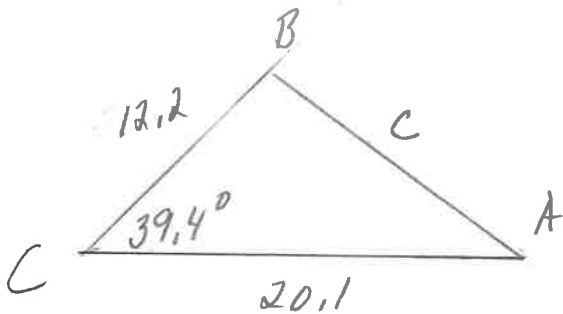
Given any triangle with angles at the vertices A , B , and C with corresponding opposite sides a , b , and c

$$a^2 = b^2 + c^2 - 2bc \cos A$$



Example: (SAS)

Solve the triangle ABC where $C = 39.4^\circ$, $a = 12.2$, $b = 20.1$



$$c^2 = (12.2)^2 + (20.1)^2 - 2(12.2)(20.1) \cos(39.4)$$

$$c^2 = 148.84 + 404.01 - 378.98$$

$$c^2 = 173.87, \quad \boxed{c = 13.186}$$

$$a^2 = b^2 + c^2 - 2ab \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2ab}, \quad \cos A = .8748$$

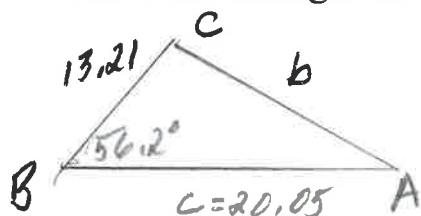
$$\boxed{A = 28.979^\circ}$$

$$B = 180 - 39.4 - 28.979$$

$$\boxed{B = 111.62^\circ}$$

Example: (SAS)

Solve the triangle ABC where $B = 56.2^\circ$, $a = 13.21$, $c = 20.05$



$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = (13.21)^2 + (20.05)^2 - 2(13.21)(20.05) \cos 56.2^\circ$$

$$b^2 = 281.825, \quad \boxed{b = 16.788}, \quad a^2 = b^2 + c^2 - 2abc \cos A$$

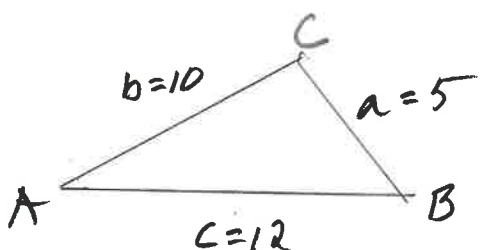
$$(13.21)^2 = (16.788)^2 + (20.05)^2 - 2(16.788)(20.05) \cos A$$

$$\cos A = \frac{(16.788)^2 + (20.05)^2 - (13.21)^2}{2(16.788)(20.05)} = .7566$$

$$\boxed{A = 40.84^\circ}, \quad C = 180 - 40.84 - 56.2, \quad \boxed{C = 82.96^\circ}$$

Example: (SSS)

Solve the triangle ABC where $a = 5$, $b = 10$, $c = 12$



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$144 = 25 + 100 - 100 \cos C$$

$$\cos C = \frac{25 + 100 - 144}{100} = -.19$$

$$\boxed{C = 100.953^\circ}$$

$$\frac{\sin B}{b} = \frac{\sin C}{c} \quad (\text{Law of Sines})$$

$$\sin B = \frac{10(\sin 100.953)}{12} = .8182$$

$$\boxed{B = 54.9^\circ}, \quad A = 180 - 100.953 - 54.9, \quad \boxed{A = 24.147^\circ}$$