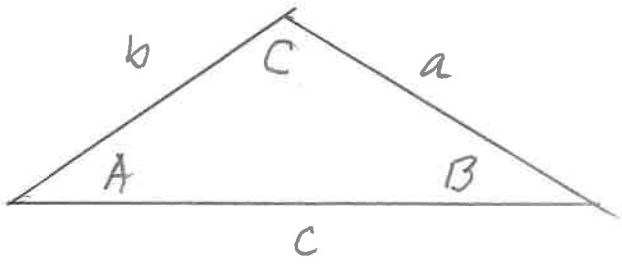
The Law of Sines

The Law of Sines:
Given any triangle with angles at the vertices $A, B,$ and C with corresponding opposite sides $a, b,$ and c

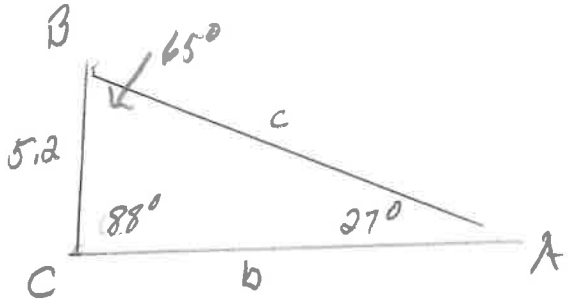
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Example: (AAS)
Solve the triangle ABC where $A = 27^o$, $C = 88^o$, $a = 5.2$



$\frac{c}{\sin 88} = \frac{5.2}{\sin 27^\circ}$

$c = \frac{(5.2)\sin 88}{\sin 27}$

$c = 11.45$

$B = 180 - 88 - 27$

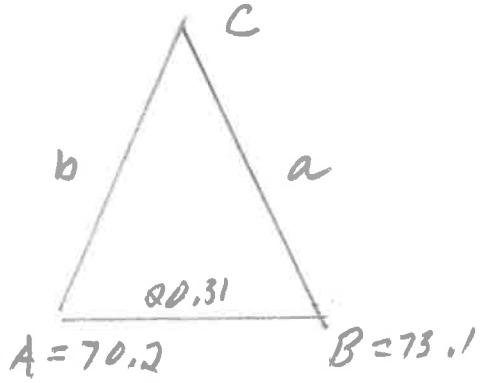
$B = 65^\circ$

$\frac{b}{\sin B} = \frac{5.2}{\sin 27^\circ}$

$b = \frac{(5.2)\sin 65^\circ}{\sin 27^\circ}$

$b = 10.38$

Example: (ASA)
Solve the triangle ABC where $A = 70.2^o$, $B = 73.1^o$, $c = 20.31$



$C = 180 - 70.2 - 73.1$

$C = 36.7^o$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$b = (20.31)\frac{\sin(73.1)}{\sin(36.7)}$

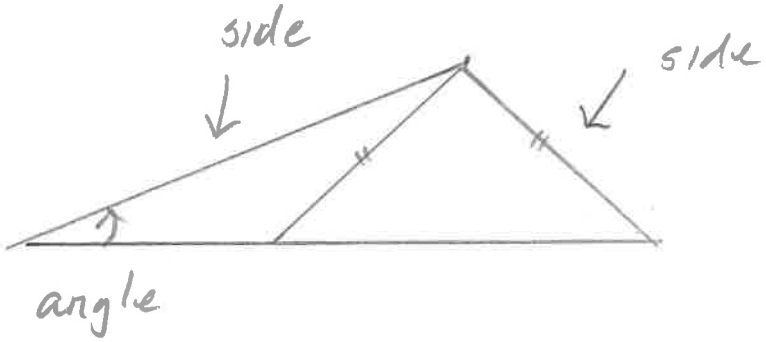
$b = 32.92$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

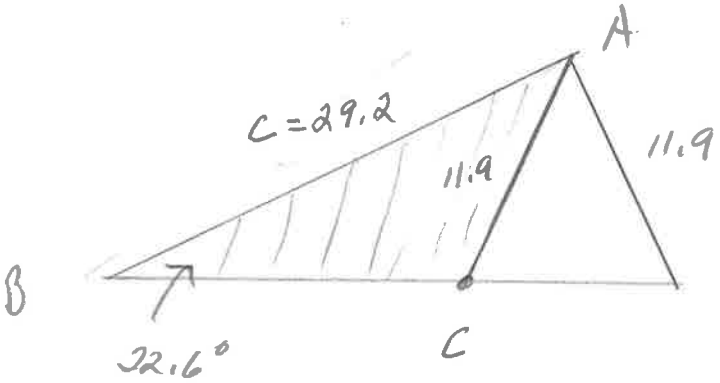
$a = (20.31)\frac{\sin(70.2)}{\sin(36.7)}$

$a = 31.98$

The Ambiguous case: (SSA)



Example: (SSA)
Solve the triangle ABC where $B = 22.6^o$, $b = 11.9$, $c = 29.2$



Case 1 : angle C is > 90°

$\frac{\sin C}{29.2} = \frac{\sin B}{b}$, $\sin C = \sin(22.6)\left(\frac{29.2}{11.9}\right)$

$\sin C = .9429$, $C = 70.558^\circ$ or

$C = 180 - 70.558^\circ$, $C = 109.442^\circ$

$A = 180 - 109.442 - 22.6$, $A = 47.958^\circ$

$\frac{a}{\sin A} = \frac{b}{\sin B}$, $a = (11.9)\left(\frac{\sin 47.958}{\sin(22.6)}\right)$

$a = 22.997$

Case 2 : $C = 70.558^\circ$, $A = 180 - 22.6 - 70.558$

$A = 86.842^\circ$, $\frac{a}{\sin A} = \frac{b}{\sin B}$, $a = (11.9)\frac{\sin A}{\sin B}$

$a = 30.919$