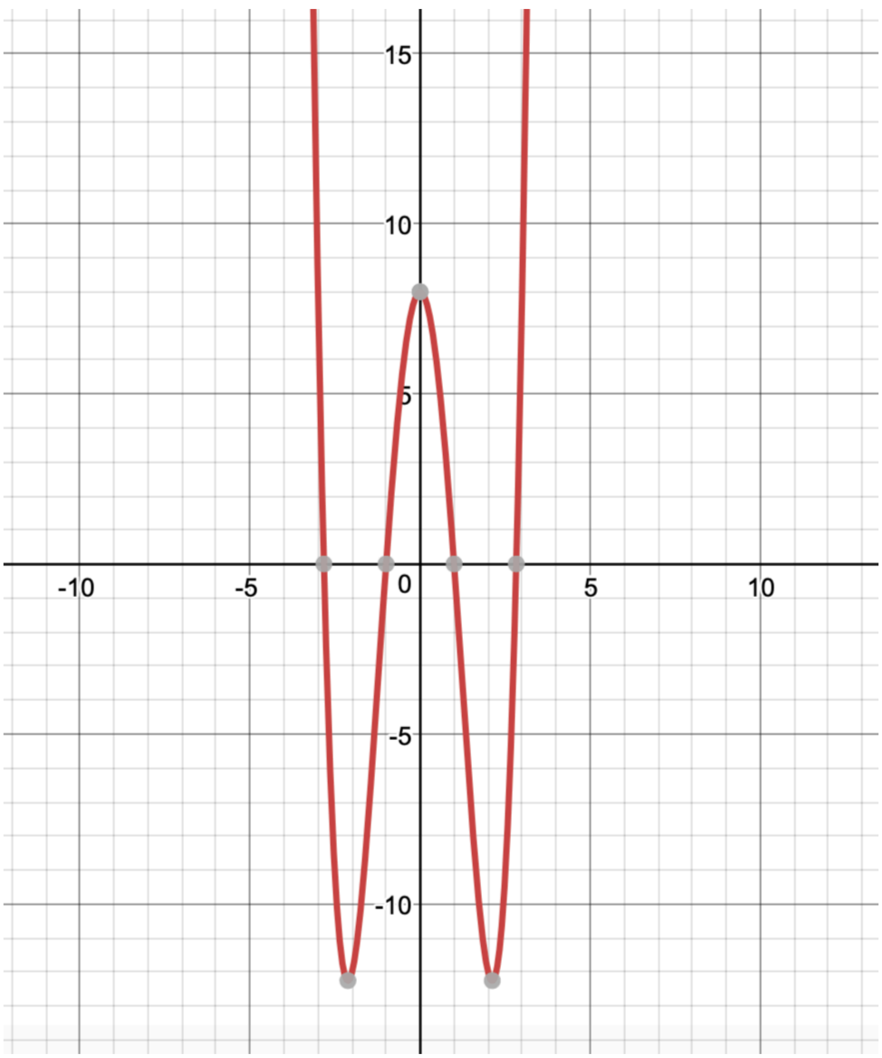
Transformations of Functions ($Part\ 2$)
Symmetry:

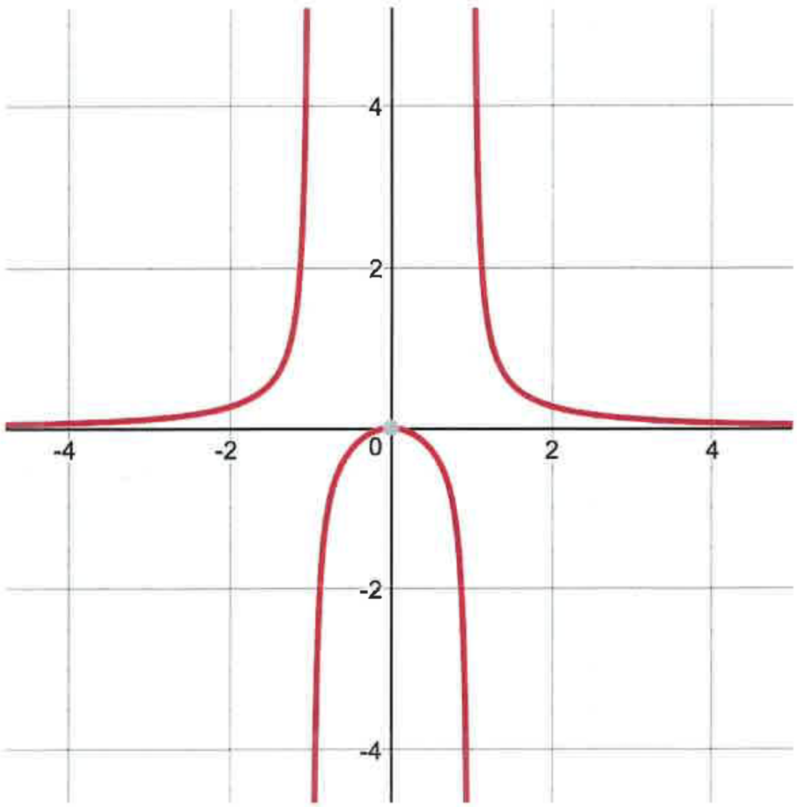
y-axis Symmetry:

The graph of a function f is **symmetric with respect to the y-axis** if $f(-x) = f(x)$ for all x in the domain of f. A function that is symmetric with respect to the y-axis is called an **even** function.

Examples of even functions:



$$f(x) = x^4 - 9x^2 + 8$$

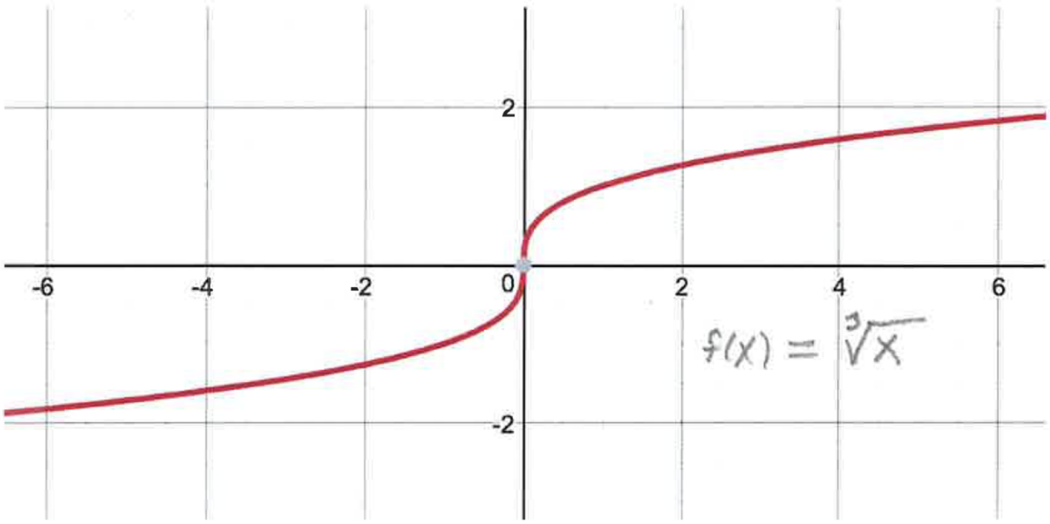


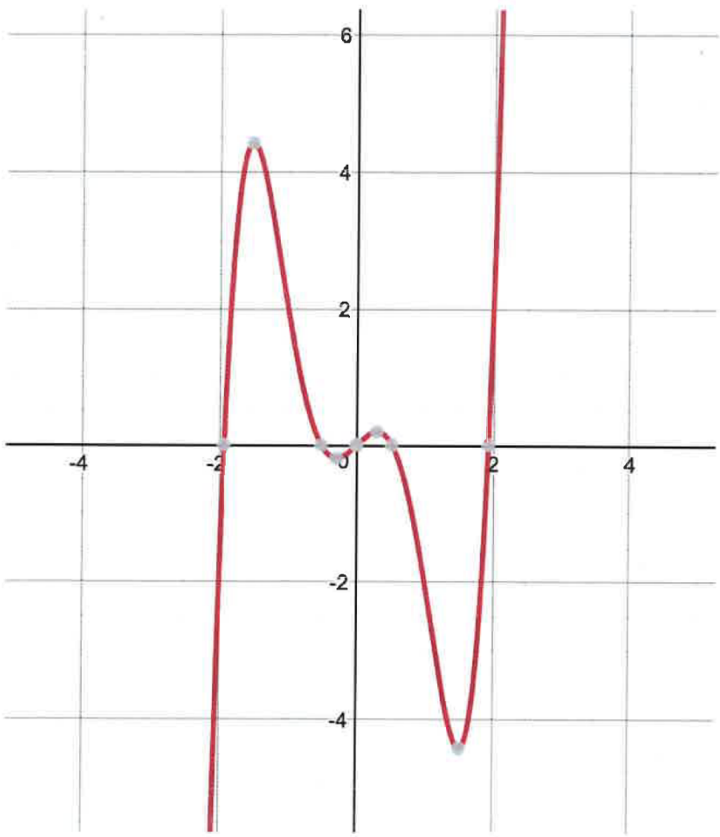
$$f(x) = \frac{x^2}{x^4 - 1}$$

Origin Symmetry:

The graph of a function f is **symmetric with respect to the origin** if $f(-x) = -f(x)$ for all x in the domain of f. A function that is symmetric with respect to the origin is called an **odd** function.

Examples of odd functions:





$g(x) = x^5 - 4x^3 + x$

Monotonicity:

A function f is said to be **increasing** on an interval if for all $x_1 < x_2$ then $f(x_1) < f(x_2)$.

A function f is said to be **decreasing** on an interval if for all $x_1 < x_2$ then $f(x_1) > f(x_2)$.

A function f is said to be **constant** on an interval if for all $x_1 < x_2$ then $f(x_1) = f(x_2)$.

A function f is said to be **monotone** on an interval if f is increasing, decreasing, or constant on the entire interval.

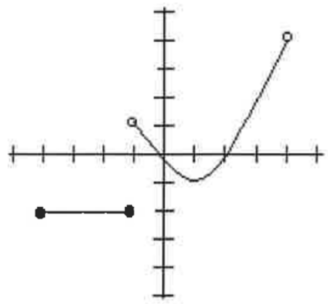
Determine the intervals of monotonicity of the function
$f(x) = -(x+3)^2 + 1$

Quadratic, the graph is a parabola with vertex at $(-3, 1)$. It opens downward & has a maximum value of 1 when $x = -3$.

f is increasing on $(-\infty, -3]$

f is decreasing on $(-3, \infty)$

Consider the function g whose graph is shown below:



What is the domain of g $[-4, 4)$

What is the range of g $\{-2\} \cup [-1, 4)$

What is $g(1)$ $= -1$

What is $g(-3)$ $= -2$

What is $g(-2)$ $= -2$

Determine the intervals of monotonicity of g.

g is decreasing on $(-1, 1]$, g is increasing on $[1, 4)$

g is constant on $[-4, -1]$