# Section 1.8 Solving Rational and Radical Equations

**Solving Rational Equations**:

1. Identify the any values that must be excluded from the solutions set because of division by zero.

2. Factor all polynomials in the numerator or denominator of any fraction. Use the factors of the denominators to determine the least common multiple of the denominators.

3. Multiply both sides of the equation by the least common multiple to write the expression with no fractions.

4. Continue to simplify and solve the equation.

Example:
Solve the equation: $\frac{3}{2x^2-2} - \frac{1}{x+1} = \frac{1}{6}$

$$\frac{3}{2(x^2-1)} - \frac{1}{(x+1)} = \frac{1}{6}$$

$$\frac{3}{2(x+1)(x-1)} - \frac{1}{(x+1)} = \frac{1}{6},$$ multiply by $6(x+1)(x-1)$

$$9 - 6(x-1) = (x+1)(x-1)$$

$$9 - 6x + 6 = x^2 - 1$$

$$x^2 - 1 + 6x - 15 = 0$$

$$x^2 + 6x - 16 = 0$$

$$(x+8)(x-2) = 0$$

$x = -8$ or $x = 2$

Example:

Solve the equation: $\frac{1}{2x} + \frac{2}{x+2} = \frac{3}{4x}$

Multiply both sides by $4x(x+2)$

$$2(x+2) + 8x = 3(x+2)$$

$$2x + 4 + 8x = 3x + 6$$

$$10x + 4 = 3x + 6$$

$$7x = 2$$

$$x = \frac{2}{7}$$

Example:
Solve the equation: $2 - \frac{3}{x} = \frac{3}{x+1}$

multiply both sides by $x(x+1)$

$$2x(x+1) - 3(x+1) = 3x$$

$$2x^2 + 2x - 3x - 3 = 3x$$

$$2x^2 - x - 3 = 3x$$

$$2x^2 - 4x - 3 = 0$$

Quadratic formula:

$$x = \frac{4 \pm \sqrt{16 - 4(2)(-3)}}{4}$$

$$x = \frac{4 \pm \sqrt{16 + 24}}{4} = \frac{4 \pm \sqrt{40}}{4}$$

$$x = \frac{4 \pm 2\sqrt{10}}{4} = 1 \pm \frac{\sqrt{10}}{2}$$

## Solving Equations With Radicals:

1. Isolate the radical and then raise both sides of the equation to the appropriate power. This will often result in an equation with extraneous solutions.

2. If there is more than one radical, isolate one radical at a time.

3. Check your solutions when you are done.

Example:
Solve the equation $\sqrt{2x+2} - x + 3 = 0$

$\sqrt{2x+2} = x - 3$

$2x + 2 = x^2 - 6x + 9$

$x^2 - 8x + 7 = 0$

$(x-7)(x-1) = 0$

$x = 7, \quad x = 1$

$x = 7$ is a solution

$x = 1$ is not a solution.

Example:

Solve the equation $\sqrt{x} + \sqrt{x - 20} = 10$

$$x + 2\sqrt{x}\sqrt{x-20} + x - 20 = 100$$

$$2\sqrt{x}\sqrt{x-20} = -2x + 120$$

$$\sqrt{x}\sqrt{x-20} = -x + 60$$

$$x(x-20) = x^2 - 120x + 3600$$

$$\cancel{x^2} - 20x = \cancel{x^2} - 120x + 3600$$

$$100x = 3600$$

$$x = 36$$

check: $\sqrt{36} + \sqrt{16} \stackrel{?}{=} 10$

$$6 + 4 = 10$$

ok

$x = 36$ is a solution.

Example:

Solve the equation $(x^2+21)^{-3/2} = \frac{1}{125}$

$$x^2+21 = \left(\frac{1}{125}\right)^{-\frac{2}{3}}$$

$$x^2+21 = \left(\frac{1}{5}\right)^{-2} = \frac{1}{5^{-2}} = \frac{1}{1/5^2}$$

$$x^2+21 = \frac{1}{1/25} = 25$$

$$x^2 = 4 \qquad x = \pm 2$$