Solving Equations

Definition:
An **equation** is an expression that contains an equal sign. An equation can be true or false.

An equation that is true is always used to describe a relationship between two expressions that are the same expression.

We assume that a given equation is true unless we can show it to be false.

Examples of Equations:

$3 + 2 = 5$ (Identity)

$3 + 2 = 7$ (Contradiction)

$3 + x = 7$ (Conditional) True only for $x = 4$

$3(x + 2) = 3x + 6$ (Identity)

True for all x.

When we solve an equation we are determining the values for any unkowns that satisfy the requirement that equal expressions represent the same numeric quantity.

When we solve an equation we are determining the possible values for any unknowns that will make the equation true. All such values form a set of numbers called the **solution set**. When you solve an equation you are trying to determine the solution set for a given equation.

Solve:

$5x = 3$ $x = \frac{3}{5}$ solution set is $\{\frac{3}{5}\}$

Solve:

$x + 2 = 3$ $x = 1$ Formally, the solution set is $\{1\}$.

Solve:

$2(2x - 3) + 3x - 4 = 3(x + 1) - 2x + 1$

$4x - 6 + 3x - 4 = 3x + 3 - 2x + 1$

$7x - 10 = x + 4$

$6x = 14, \quad x = \frac{14}{6}, \quad x = \frac{7}{3}$

$\{\frac{7}{3}\}$

Solve the equation below:

$\frac{x+3}{2} + x = \frac{2x-1}{4}$, multiply both sides by 4

$2x+6+4x = 2x-1$

$6x+6 = 2x-1$

$4x = -7$

$x = -\frac{7}{4}$ $\left\{-\frac{7}{4}\right\}$

Use a different set of simplification steps to solve the equation:

$\frac{x+3}{2} + x = \frac{2x-1}{4}$

$\frac{x}{2} + \frac{3}{2} + x = \frac{x}{2} - \frac{1}{4}$

$\frac{3}{2} + x = -\frac{1}{4}$

$x = -\frac{1}{4} - \frac{3}{2} = -\frac{1}{4} - \frac{6}{4} = -\frac{7}{4}$

$\left\{-\frac{7}{4}\right\}$

Solve the equation below:

$2x - 6 = 2(x - 4)$

$2x - 6 = 2x - 8$

$-6 \neq -8$

No solution

Solve the equation below:

$2x + 8 - \frac{2}{3} = \frac{2}{3}(3x + 11)$

$6x + 24 - 2 = 6x + 22$

$6x + 22 = 6x + 22$

identity, true for all x

solution set is $\{x : -\infty < x < \infty\}$