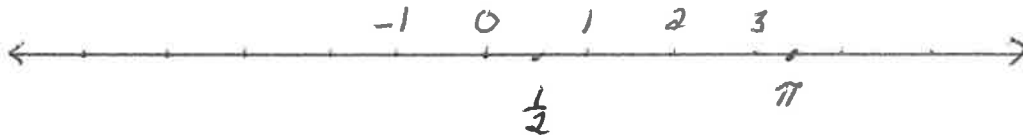# Real Numbers and Algebraic Expressions

Definition of real numbers •
The real numbers are the numbers that have a one-to-one correspondence with the points on a real number line.

What are some examples of real numbers?

My favorite: $3, 0, \pi, \sqrt{2}, e$
$75, -32, 19, 1943$

What are some numbers that are not real numbers?

complex numbers:
$1 + i, \quad \sqrt{-1} = i$
$-3 + \sqrt{5}\, i,$
$\mathbb{C} = \{ a + bi : a \in \mathbb{R}, b \in \mathbb{R} \}$

Subsets of the real numbers:

Natural Numbers:

$$J = \{1, 2, 3, 4, \text{...}\}$$

Whole Numbers:

$$W = \{0, 1, 2, 3, 4, \text{...}\}$$

Integers:

$$Z = \{\text{...} -3, -2, -1, 0, 1, 2, 3, \text{...}\}$$

Rational Numbers:

$$Q = \left\{ \frac{a}{b} : a \in Z, b \in Z, b \neq 0 \right\}$$

Irrational Numbers:

$$I = \{x : x \in R, x \notin Q\}$$

examples: $\sqrt{2}, -\sqrt{7}, \pi, -4\pi, e$

$$N \subseteq W \subseteq Z \subseteq Q$$

$$R = Q \cup I$$

Interval Notation:

$(2, 6) = \{ x : 2 < x < 6 \}$

$[-2, 4] = \{ x : -2 \le x \le 4 \}$

$\left(-\frac{1}{2}, \frac{13}{2}\right] = \left\{ x : -\frac{1}{2} < x \le \frac{13}{2} \right\}$

$(-\infty, 4] = \{ x : x \le 4 \}$

$(-2, \infty) = \{ x : -2 < x \}$

$(-2, 4) \cup (2, 7] = \{ -2 < x < 4 \quad \text{or} \quad 2 < x \le 7 \}$
$= \{ x : -2 < x \le 7 \}$

$(-2, 4) \cap (2, 7] = \{ x : -2 < x < 4 \quad \text{and} \quad 2 < x < 7 \}$
$= \{ x : 2 < x < 4 \}$

Absolute Value:

$$|x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

Simplify the following

$|2-5| \quad = \quad |-3| = 3$

$-|3+4| \quad = \quad -|7| = -7$

$|3 - |2-5|| \quad = \quad |3 - |-3|| = |3+3| = 6$

$|x| \quad = \quad$ distance from x to 0.

$|a-b| =$ distance from a to b.

Order of Operations $(PEMDAS)$

Evaluate the following $2 + 3 \div (4+2)\,3 - 2$

$$= \quad 2 + \frac{3}{18} - 2 \quad = \quad \frac{1}{6}$$

Evaluate the following using a calculator:

$$\frac{2 + \frac{5}{2.12+3.14} - 2.12 \times 4.21}{\sqrt{4.15 - 2.67} + 2.143} \quad = \quad \frac{-5.97462965}{3.35955250}$$

$$= \quad -1.7784$$

Field Properties of Algebra:

Name Of Property	Additive Version	Multiplicative Version
Closure	$a + b$ is a real number	ab is a real number
Commutative	$a + b = b + a$	$ab = ba$
Associative	$a + (b + c) = (a + b) + c$	$a(bc) = (ab)c$
Identity	$a + 0 = a$	$a(1) = a$
Inverse	$a + (-a) = 0$	$a\left(\frac{1}{a}\right) = 1$ for $a \neq 0$
Distributive	$a(b + c) = ab + ac$	$a(b + c) = ab + ac$

Zero Factor Property:

If $AB = 0$ then $A = 0$ or $B = 0$.

Cancellation Properties:

If $A = B$ then $A + C = B + C$
Adding the same quantity to both sides of an equation results in an equivalent equation.

For $C \neq 0$, If $A = B$ then $AC = BC$
Multiplying both sides of an equation by the same non-zero quantity results in an equivalent equation.