Solving Exponential and Logarithmic Equations

An **exponential equation** is any equation where the variable to be solved for is found in an exponent of the expression.

Review:

Solve the equation elementary exponential equation $3^{x+2} = 27^{x-1}$

$$(x+2)\log_3 3 = (x-1)\log_3 27$$

$$(x+2) = (x-1)3, \quad x+2 = 3x-3$$

$$-2x = -5, \quad x = \frac{5}{2}$$

Solve the equation $9^{x+1} = 1237$

$$(x+1)\ln 9 = \ln(1237)$$

$$x+1 = \frac{\ln(1237)}{\ln 9}$$

$$x = \frac{\ln(1237)}{\ln 9} - 1$$

$$x \approx 2.24065$$

Solve the equation $3e^{-x-8} = 27$

$$e^{-x-8} = 9$$

$$(-x-8)\ln e = \ln 9$$

$$-x-8 = \ln 9$$

$$x = -8 - \ln 9$$

$$x \approx -10.1972$$

Solve the equation $e^{2x-6} = 7^{x/10}$

$$(2x-6)\ln e = \frac{x}{10}\ln 7$$

$$2x-6 = (.194591)x$$

$$2x - (.194591)x = 6$$

$$x(2-.194591) = 6$$

$$x = \frac{6}{2-.194591} = 3.3233$$

Solve the equation $\log_{11}(x+3) - \log_{11}(x-2) = 2$

$$\log_{11}\left(\frac{x+3}{x-2}\right) = 2$$

$$11^2 = \frac{x+3}{x-2}, \quad 121(x-2) = x+3$$

$$121x - 242 = x + 3$$

$$120x = 245$$

$$x = \frac{245}{120} = \frac{49}{24}$$

Solving Logarithmic Equations:

1. Combine all logarithms into one logarithm using logarithmic properties.
2. Convert from logarithmic form to exponent form.
3. Solve for the variable.
4. Check your solutions to see if they are in the domain of the original equation!!

Solve the equation $\log_6(x-5)+\log_6(x+2)=log_6(x-2)$

$$\log_6[(x-5)(x+2)] = \log_6(x-2)$$

$$(x-5)(x+2) = (x-2)$$

$$x^2-3x-10 = x-2$$

$$x^2-4x-8 = 0$$

$$x = \frac{4 \pm \sqrt{16-4(-8)}}{2} = \frac{4\pm\sqrt{48}}{2}$$

$$= \frac{4\pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$$

$$x = 2+2\sqrt{3}$$

Solve the equation $2\,ln\,(x-3)=ln\,(3x)$

$$\ln(x-3)^2 = \ln(3x)$$

$$(x-3)^2 = 3x$$

$$x^2-6x+9 = 3x$$

$$x^2-9x+9 = 0$$

$$x = \frac{9 \pm \sqrt{81-4(9)}}{2} = \frac{9\pm\sqrt{45}}{2}$$

$$x = \frac{9\pm 3\sqrt{5}}{2}, \quad x = \frac{9+3\sqrt{5}}{2}$$

Solve the equation $\ln(x-4) - \ln x = 3$

$$\ln\left(\frac{x-4}{x}\right) = 3, \quad \frac{x-4}{x} = e^3$$

$$x-4 = e^3 x, \quad e^3 x - x = -4$$

$$x(e^3-1) = -4, \quad x = \frac{-4}{e^3-1}$$

$\frac{-4}{e^3-1} < 0$, not in the domain of the equation. So, no solution.

Steve is saving up money for a down payment on a motorcycle. He currently has \$4187, but knows he can get a loan at a lower interest rate if he can put down \$4903. If he invests the \$4187 in an account that earns 5.1% annually, compounded monthly, how long will it take Steve to accumulate the \$4903?

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

$$4903 = 4187\left(1+\frac{.051}{12}\right)^{12t}$$

$$\frac{4903}{4187} = (1.00425)^{12t}$$

$$\ln\left(\frac{4903}{4187}\right) = 12t \ln(1.00425)$$

$$t = \frac{\ln\left(\frac{4903}{4187}\right)}{12\ln(1.00425)} = 3.102 \text{ years}$$