Properties of Logarithms

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If b is a positive real number then the following relationships exist:

1. $log_b 1 = 0$
2. $log_b b = 1$
3. $log_b b^x = x$
4. $b^{log_b x} = x$ for $x > 0$ (Why do we have the limitation $x > 0$?)

5. $log_b(MN) = log_b M + log_b N$ for $M > 0$ and $N > 0$
6. $log_b(\frac{M}{N}) = log_b M - log_b N$ for $M > 0$ and $N > 0$
7. $log_b(M^p) = p\, log_b M$ for $M > 0$

Simplify the following logarithmic expressions without using a calculator:

$log_2 8\sqrt{2}$ $= log_2 8 + log_2 \sqrt{2} = 3 + \frac{1}{2} = 3.5$

$log_5 \frac{\sqrt[3]{5}}{25}$ $= log_5 (5)^{\frac{1}{3}-2} = \frac{1}{3} - 2 = -\frac{5}{3}$

$log_9 3 + log_9 27$ $= log_9 (3 \cdot 27) = log_9 9^2 = 2$

$e^{-4\,ln\,2 + ln\,3}$ $= e^{-4\,ln\,2} \cdot e^{ln\,3} = e^{ln(2^{-4})} \cdot e^{ln\,3}$

$= 2^{-4} \cdot 3 = 3/16$

Use the properties of logarithms to write the following as a logarithm of a single quantity:

$4\,log_2\,(xy^2) + 2\,log_2\,\left(\frac{1}{x}\right) - 4\,log_2 y$

$= \log_2 (xy^2)^4 + \log_2 \left(\frac{1}{x}\right)^2 - \log_2 (y)^4$

$= \log_2 \dfrac{(xy^2)^4 \cdot \left(\frac{1}{x}\right)^2}{y^4}$

$= \log_2 \dfrac{x^4 y^8}{x^2 y^4} = \log_2 \dfrac{x^2}{y^4}$

$ln\,7 + 6\,ln\,x - 6\,ln\,y \quad = \quad \ln 7 + \ln(x^6) - \ln(y)^6$

$= \ln \dfrac{7 \cdot x^6}{y^6}$

Expand the following logarithms:

$log_5(50x + 25y) \quad = \log_5 25(2x+y)$

$= \log_5 25 + \log_5 (2x+y)$

$= 2 + \log_5 (2x+y)$

$log_2(8x^2 + 80x + 200)$

$= \log_2 8(x^2+10x+25)$

$= \log_2 8 + \log_2 (x^2+10x+25)$

$= 3 + \log_2 (x^2+10x+25)$

$log\,(\,log\,(10000^{250x}\,)) \quad = \log[250x \log 10000]$

$= \log[250x \cdot 4] = \log[1000x]$

$= \log 1000 + \log x = 3 + \log x$

The Change of Base Formula:

If b, c, and x are positive real numbers then:

$log_b x = \frac{log_c x}{log_c b}$

Give a 4 decimal approximation for $log_3 20$.

$$log_3 20 = \frac{log_{10} 20}{log_{10} 3} = \frac{log\ 20}{log\ 3} \approx 2.7268$$

Give a four decimal approximate for the number $log_{\frac{1}{7}}(0.719)$

$$= \frac{log_{10}(.719)}{log_{10}(\frac{1}{7})} \approx +.16953$$

The **pH** of a solution is defined to be $-\, log\,([H_3O^+])$, where $[H_3O^+]$ is the concentration of hydronium ions in moles/liter. Solutions with a pH less than 7 are said to be *acidic*, while those with a pH greater than 7 are *basic*.

Example:
If a sample of a solution has a $[H_3O^+]$ concentration of 7.49×10^{-6} moles/liter. What is the pH?

$$-\log[(7.49)10^{-6}] = 5.1255$$

(acidic)

If I_0 is the minimum discernable intensity of an earthquake, then an earthquake with an intensity of I has a ***Richter Scale*** ranking of:
$R = log\left(\frac{I}{I_0}\right)$

If I_0 is the minimum discernable intensity of a sound, then a sound with an intensity of I has a **decibel level** of:
$D = 10\,log\left(\frac{I}{I_0}\right)$

Example:
Given that $I_0 = 10^{-12}$ watts/meter^2. What is the intensity of a sound for which the decibel level is 102?

$$10\log\left(\frac{102}{10^{-12}}\right) = 10\log\left(102 \cdot 10^{12}\right)$$

140.086