Exponential Functions

Before we study exponential functions, make sure to review and understand some of the properties of exponents that you learned in beginning and intermediate algebra:

$a^n \, a^m = a^{n+m}$

$\frac{a^n}{a^m} = a^{n-m}$

$(a^n)^m = a^{nm}$

$(ab)^n = a^n b^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

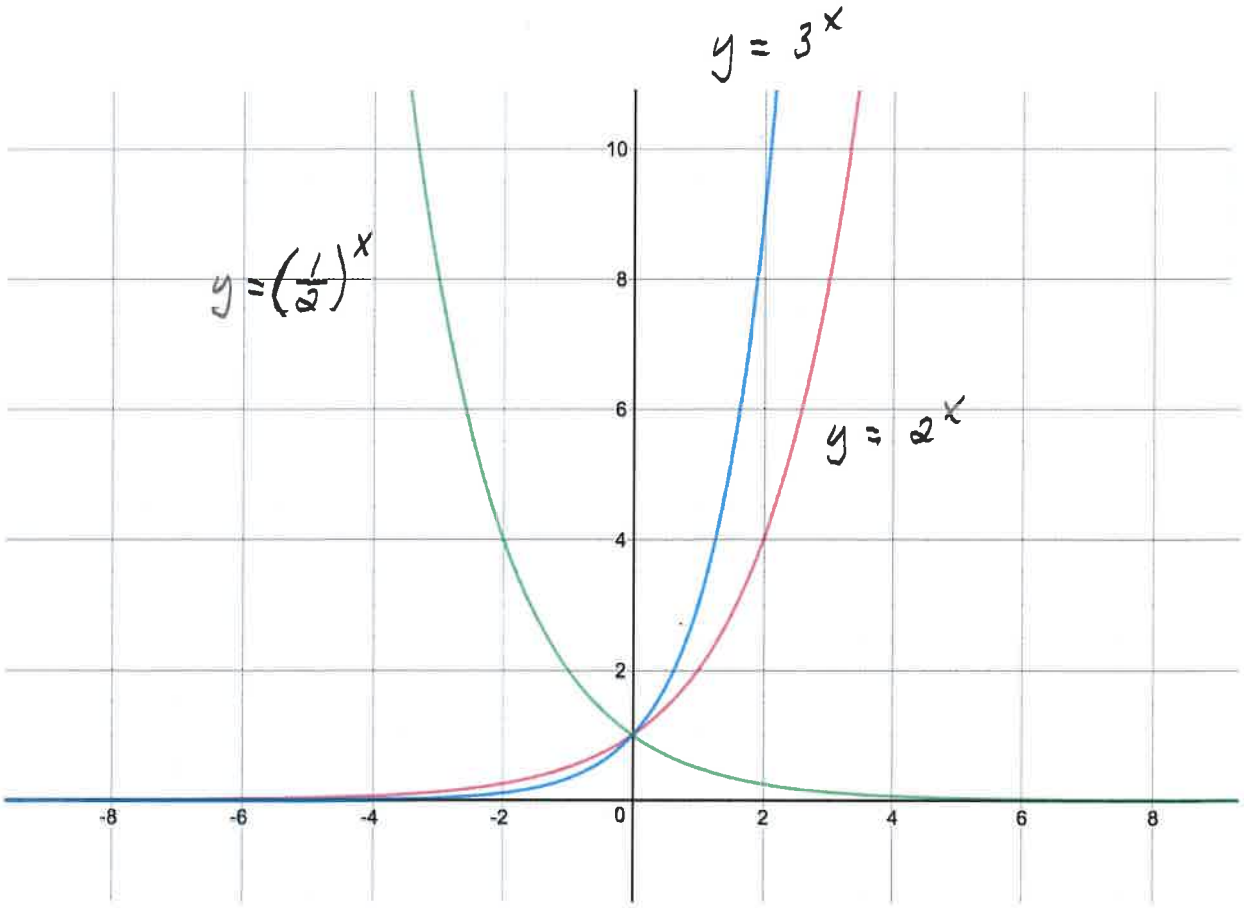
$a^0 = 1$ for all $a \neq 0$

$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$ (examples: $3^{-2} = \frac{1}{9}$; $x^{-4} = \frac{1}{x^4}$)

$a^{m/n} = (\sqrt[n]{a}\,)^m = \sqrt[n]{a^m}$ (examples: $3^{1/2} = \sqrt{3}$; $10^{2/3} = \sqrt[3]{100}$)

An **exponential** function is any function that can be written as $f(x) = a^x$ where a is any positive real number except 1.

Use the axes below and draw a graph of the functions $f(x) = 2^x$ $g(x) = 3^x$ and $h(x) = \left(\frac{1}{2}\right)^x$ by independently choosing value for x and plotting points.

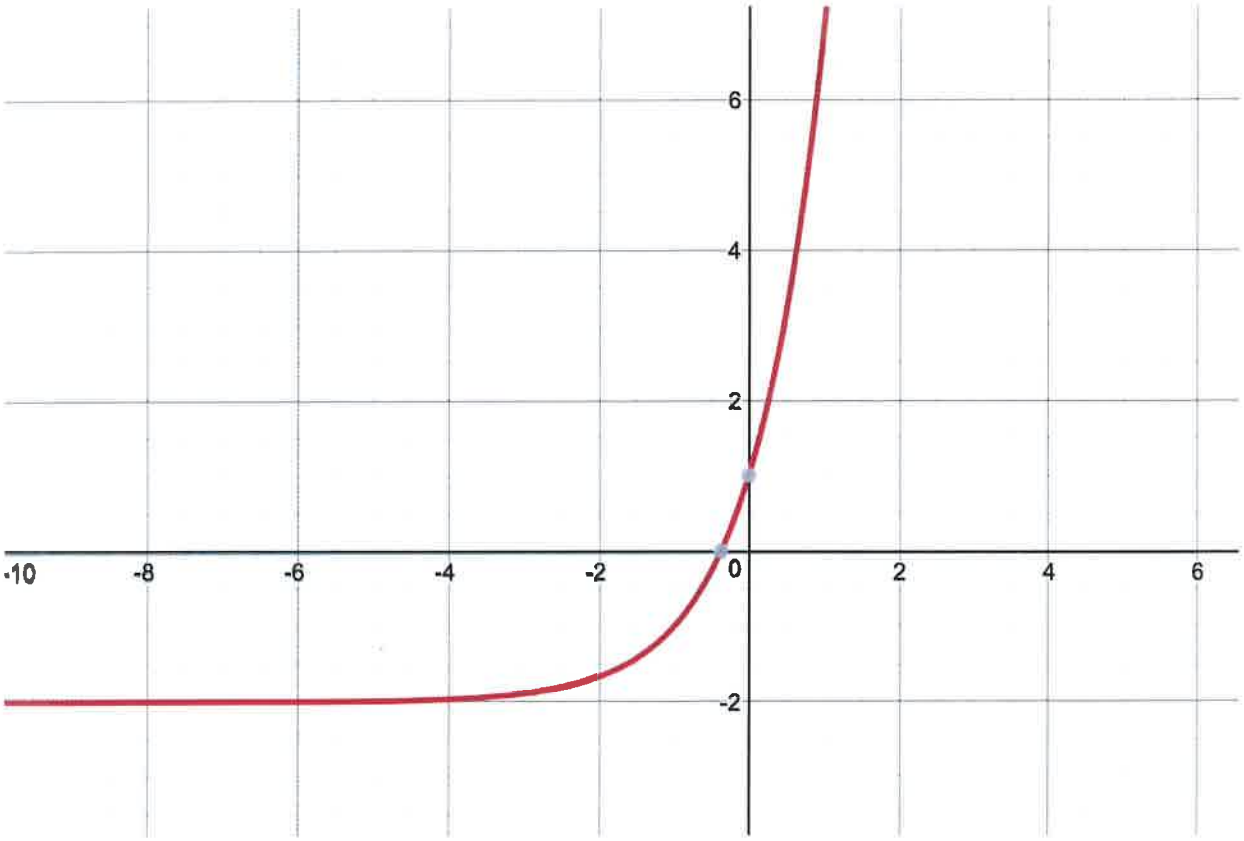


Properties of Exponential Functions

1. Graph contains the point $(0, 1)$
2. Domain all reals
3. Range $y > 0$
4. x-axis is horizontal asymptote.
5. One-to-One.
6. Increasing for $a > 1$.
   Decreasing for $0 < a < 1$.

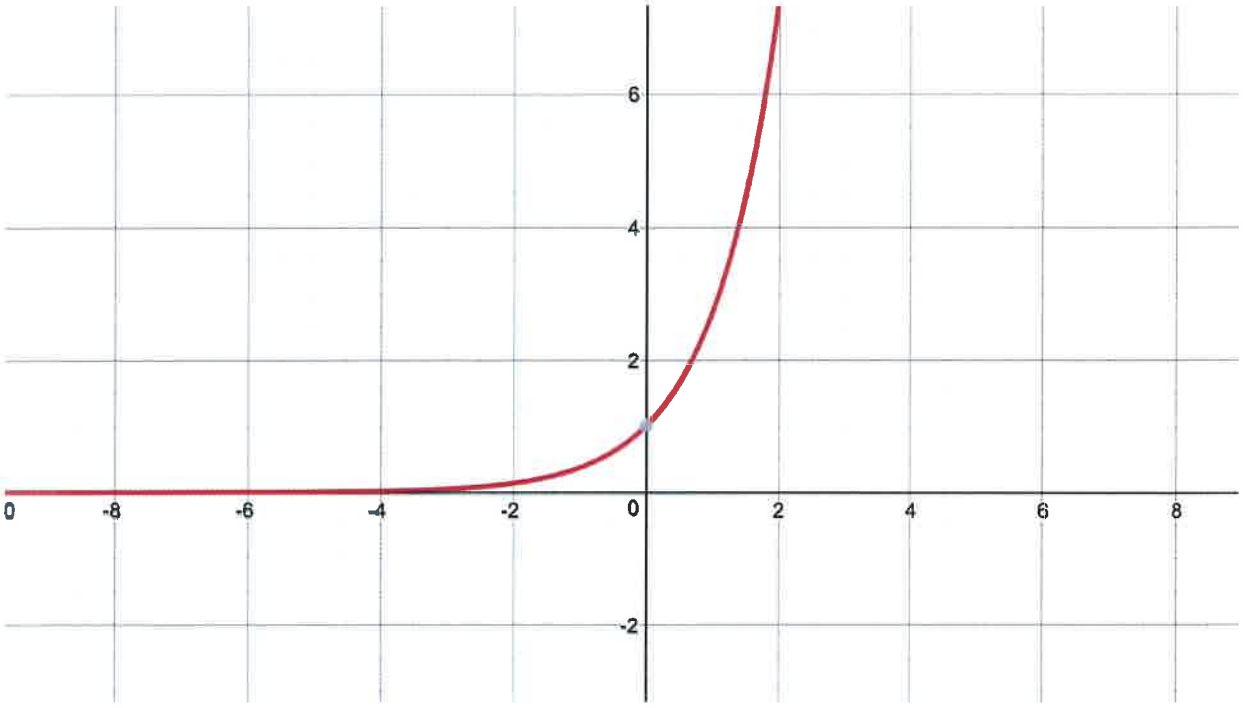
Use Transformations to write the graph of the following functions.

$y = 3^{x+1} - 2$



The number e is an irrational number called Euler's number.
$e \approx 2.718$

Use the axes below and draw a graph of the natural exponential function
$f(x) = e^x$



Solve the elementary exponential equations:

$2^{4x+1} = 8^{3x-2}$ $= (2^3)^{3x-2}$

$2^{4x+1} = 2^{9x-6}$

$4x+1 = 9x-6$

$-5x = -7$, $x = \frac{7}{5}$

$100^{3-x} = 1000^{2x+1}$

$(10^2)^{3-x} = (10^3)^{2x+1}$

$10^{6-2x} = 10^{6x+3}$

$6-2x = 6x+3$

$-8x = -3$, $x = \frac{3}{8}$

$16^{3x-1} = \left(\frac{1}{4}\right)^{1-2x}$ $(4^2)^{3x-1} = (4^{-1})^{1-2x}$

$4^{6x-2} = 4^{-1+2x}$

$6x-2 = -1+2x$

$4x = 1$, $x = \frac{1}{4}$