Locating Real Zeros of a Polynomials

The Factor Theorem:
If c is a zero of the polynomial $p(x)$ then $(x - c)$ is a factor of $p(x)$.

The Rational Zeros Theorem:
If the rational number $\frac{p}{q}$ is a zero of a polynomial, then p is a factor of the constant term and q is a factor of the leading coefficient.

Example:
Write out a list of the possible rational zeros for the polynomials given below:

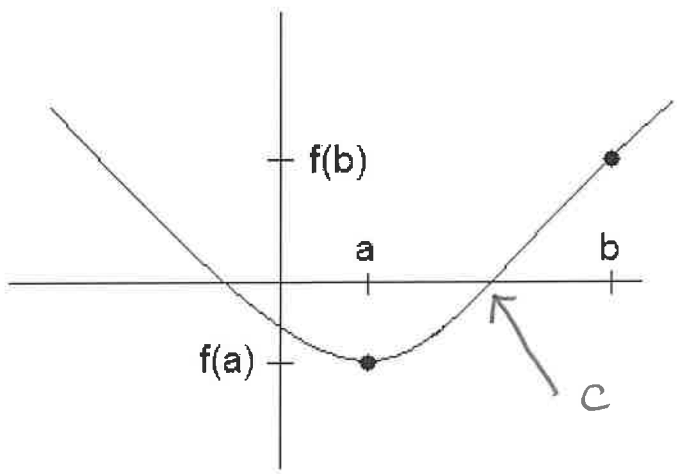
$f(x) = x^3 - 7x^2 - 4x + 28$ Factors of 28 are
$\pm 1, \pm 2, \pm 4, \pm 7$

Possible zeros are $\pm 1, \pm 2, \pm 4, \pm 7$

$g(x) = 2x^4 - 3x^3 - 6x^2 + 6x + 4$

Factors of 4 are $\pm 1, \pm 2, \pm 4$
Factors of 2 are $\pm 1, \pm 2$
Possible rational zeros are
$\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$

The Intermediate Value Theorem:
Assume that $f(x)$ is a polynomial with real coefficients, and that a and b are real numbers with $a < b$. If $f(a)$ and $f(b)$ have different signs then there is at least one real zero c of f such that $a < c < b$. *See graph below*:



$f(c) = 0$

Does the Intermediate Value Theorem guarantee that the polynomial function $f(x) = 2x^3 - x^2 - 3x$ has a zero in the interval $[1, 2]$?

$f(1) = -2, \quad f(2) = 6$

Yes!

Does the Intermediate Value Theorem guarantee that the polynomial function $g(x) = 4x^3 + 16x^2 + 5x - 25$ has a zero in the interval $[-3, -2]$?

$g(-3) = -4, \quad g(-2) = -3$

No guarantee

If $-5/2$ is a zero of g, does that contradict the Intermediate Value Theorem?

No