Polynomial Equations and Graphs

Definition:
A **polynomial function** is any function that can be written in the form
$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_o$
The **leading term** of the polynomial above is $a_n x^n$
The leading coefficient of the polynomial above is a_n
The degree of the polynomial above is n.

There are many forms a given polynomial can be written in, but two common forms are expanded form and factored form.

$x^3 - 2x^2 - 5x + 6$ is the same polynomial as $(x-1)(x-3)(x+2)$

Zeros of a function:
-If $f(k) = 0$ then k is said to be a **zero of the function** $\boldsymbol{f}$.

-If k is an input value of the function f that returns an output of 0 then c is a **zero of the function** $\boldsymbol{f}$.

-If f contains the point $(k, 0)$ then c is a **zero of the function** $\boldsymbol{f}$.

Verify that 3 is a zero of the function $f(x) = 2x^3 - 5x^2 - 9$

$f(3) = 54 - 45 - 9 = 0$

If k is a zero of the function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_o$ then we also say that k is a **root** or a **solution** of the equation $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_o = 0.$

Verify that $x = 2 + i$ is a solution to the equation $x^2 = 4x - 5$

Does $(2+i)^2 = 4(2+i) - 5$?

$4 + 4i + i^2 = 8 + 4i - 5$

$4 + i^2 = 3$

$4 + (-1) = 3$, yes

End Behaviors for the Graph of a Polynomial:

Consider the value of each term of the polynomial as x approaches infinity.

$$f(x) = -2x^3 \quad + \quad 4x^2 \quad + \quad 5x \quad + \quad 2$$

$$\begin{aligned}
f(1) &= -2 \quad + 4 \quad + 5 \quad + 2 \quad = \quad 9 \\
f(10) &= -2000 \; + \; 400 \quad + 50 \quad + 2 \quad = \quad -1548 \\
f(100) &= -2000000 + 40000 + 500 + 2 = -1959498
\end{aligned}$$

As x gets larger and larger, f(x) becomes larger & larger negatively without bound.

Consider the value of each term of the polynomial as x approaches negative infinity.

$$f(x) = -2x^3 \quad + \quad 4x^2 \quad + \quad 5x \quad + \quad 2$$

$$\begin{aligned}
f(-1) &= 2 \quad + 4 \quad - 5 \quad + 2 \quad = \quad 3 \\
f(-10) &= 2000 \; + \; 400 \quad - 50 \quad + 2 \quad = \quad 2352 \\
f(-100) &= 2000000 + 40000 - 500 + 2 = + 2039502
\end{aligned}$$

As x gets larger and larger negatively, f(x) becomes larger and larger positively without bound.

To determine the right-end behavior of a polynomial function you need to determine if the leading term of the polynomial is positive or negative when x is positive.

To determine the left-end behavior of a polynomial function you need to determine if the leading term of the polynomial is positive or negative when x is negative.

Determine the end behavior for the following polynomial functions:

$f(x) = -2x^4 - 3x^3 + 2x + 1$

As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to -\infty$

$p(x) = 4x^5 + 7x^4 - 3x^3 + 2x^2 - 10x + 4$

As $x \to \infty$, $p(x) \to \infty$

As $x \to -\infty$, $p(x) \to -\infty$

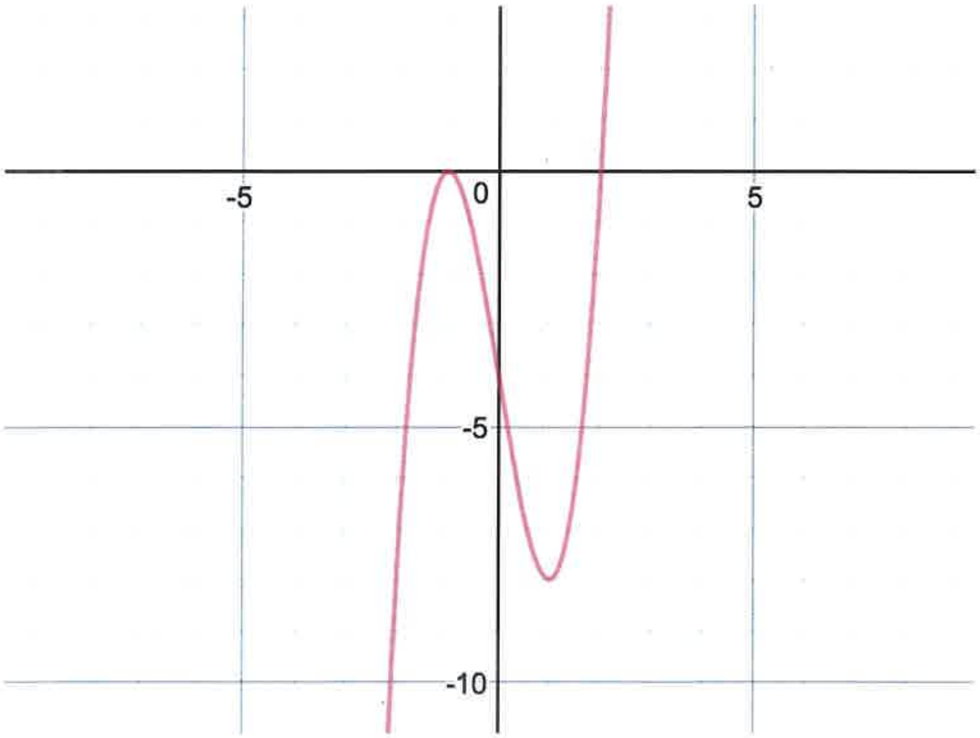
$h(x) = -\frac{2}{3}x^7 + 3x^4 - 2x^2 + 11x - 4$

As $x \to \infty$, $h(x) \to -\infty$

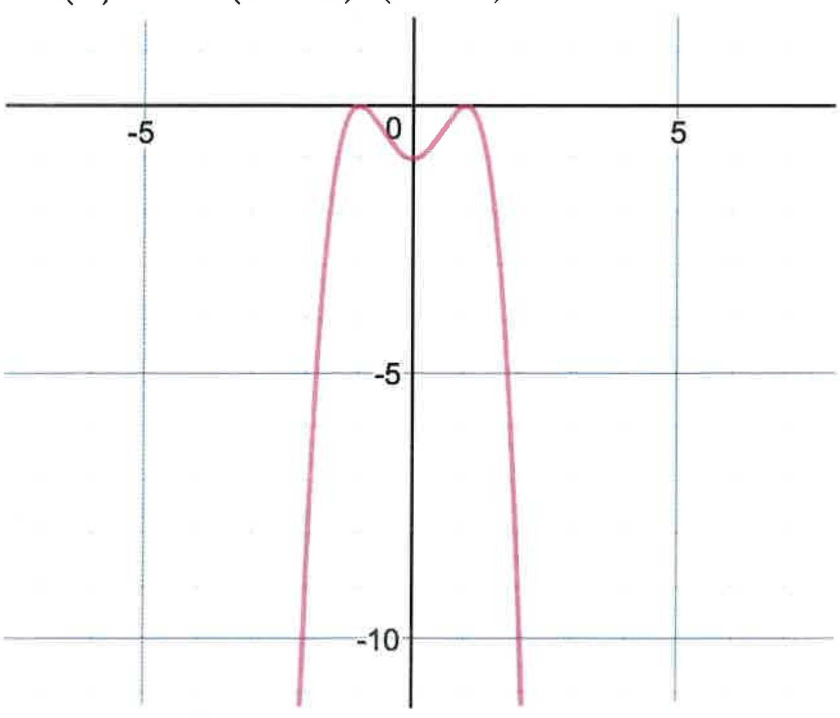
As $x \to -\infty$, $h(x) \to +\infty$

Examples:
Determine the end behaviors for the polynomial functions below and then draw the graphs of the following polynomials:

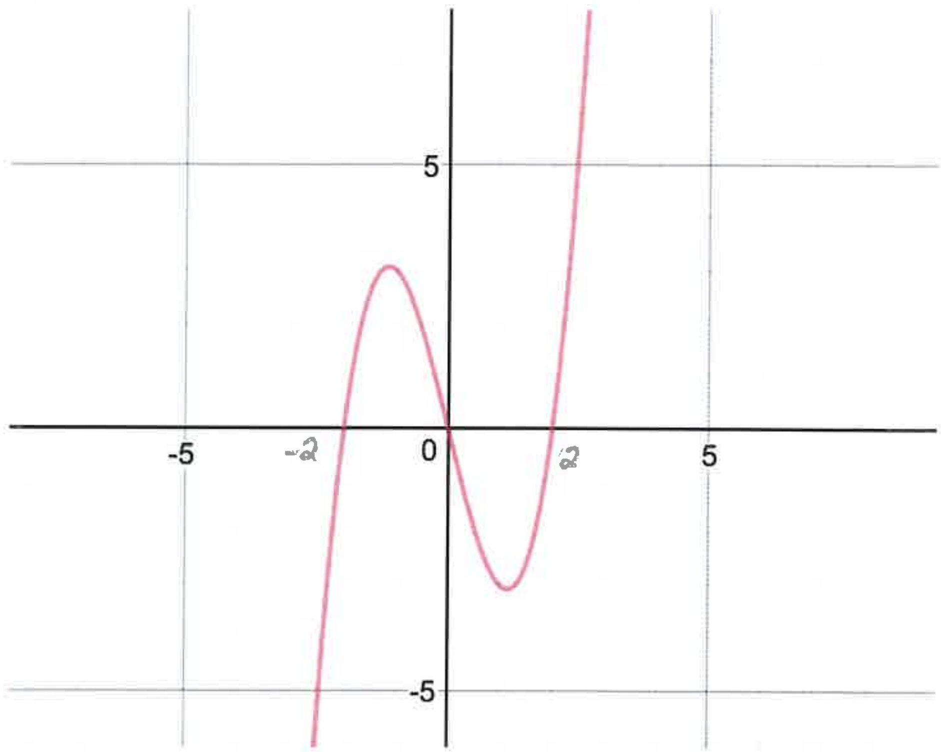
$f(x) = 2(x-2)(x+1)^2$



$h(x) = -(x+1)^2(x-1)^2$



. $p(x) = x^3 - 4x$



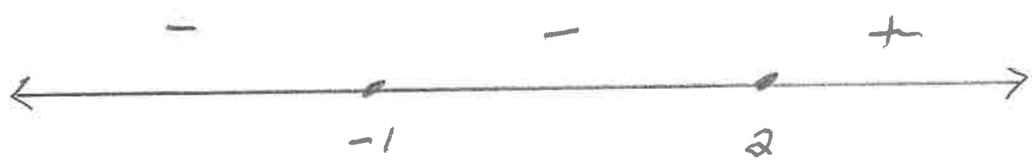
Solve the inequality $x^3 > 4x$

$x^3 - 4x > 0$, $x(x^2-4) > 0$

$x(x-2)(x+2) > 0$

$x > 2$ or $-2 < x < 0$

Solve the inequality $2(x-2)(x+1)^2 > 0$



$2 < x$