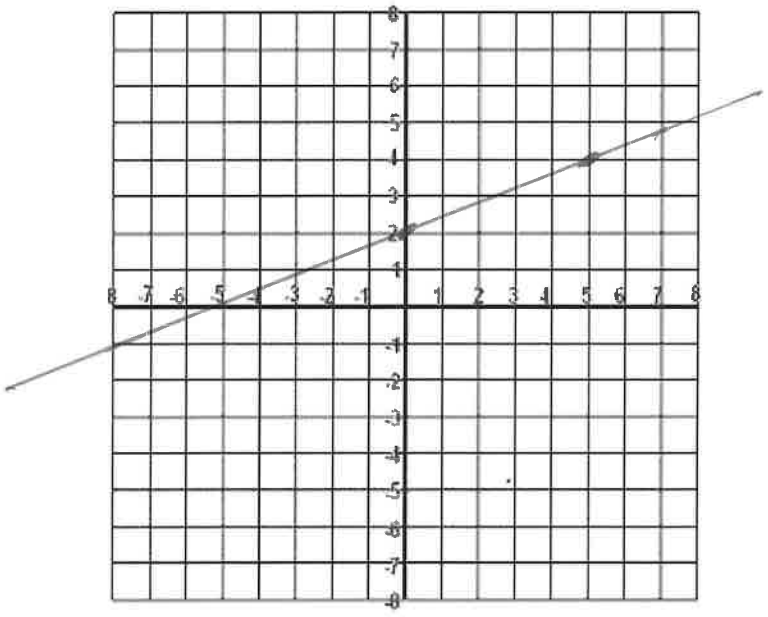
Linear and Quadratic Functions

A **linear function** f of one variable is a function where the output is determined by a linear expression. Using function notation, a **linear function** f of one variable can be written as $f(x) = mx + b$ where m and b are real numbers.

Graph the function $f(x) = \frac{2(x-1)+12}{5}$ $= \frac{2x-2+12}{5} = \frac{2x+10}{5}$



(0, 2)

(5, 4)

Definition:
A **quadratic function** is any function that can be written in the form $f(x) = ax^2 + bx + c$ where a, b, c are real numbers and $a \neq 0$.

What does the graph of a quadratic function look like?

parabola

When graphing a quadratic by hand there are 4 things I want you to determine algebraically and clearly label on the graph:

1. What are the exact coordinates of the vertex.

2. Where are the x-intercepts.

3. Where is the y-intercept.

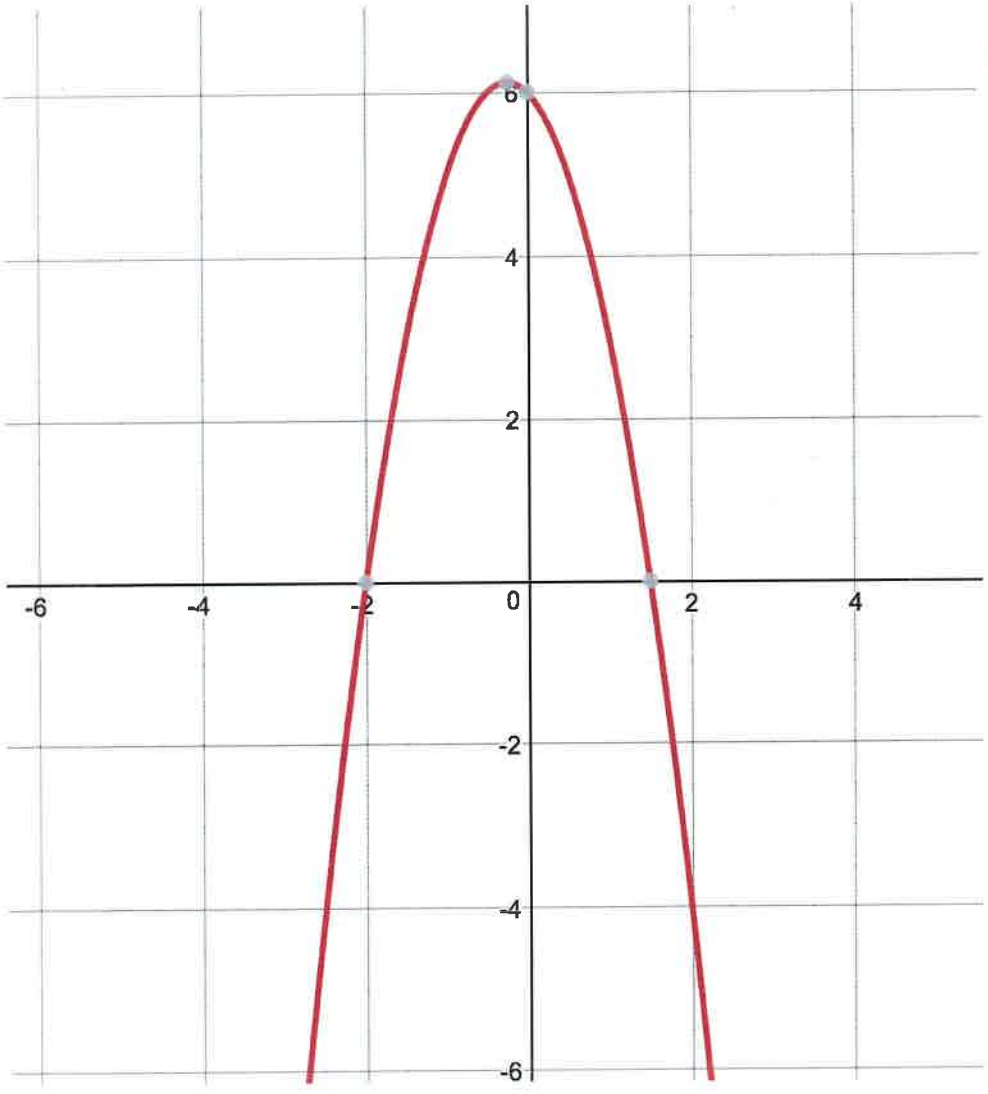
4. Two other non-intercept points on the graph.

Graph the function $h(x) = -2x^2 - x + 6$

$= -2[x^2 + x \quad] + 6$

$= -2[x^2 + x + \frac{1}{4}] + 6 + \frac{2}{4}$

$= -2(x + \frac{1}{2})^2 + \frac{13}{2}$



$(-\frac{1}{2}, \frac{13}{2})$

$(0, 6)$

y-intercept

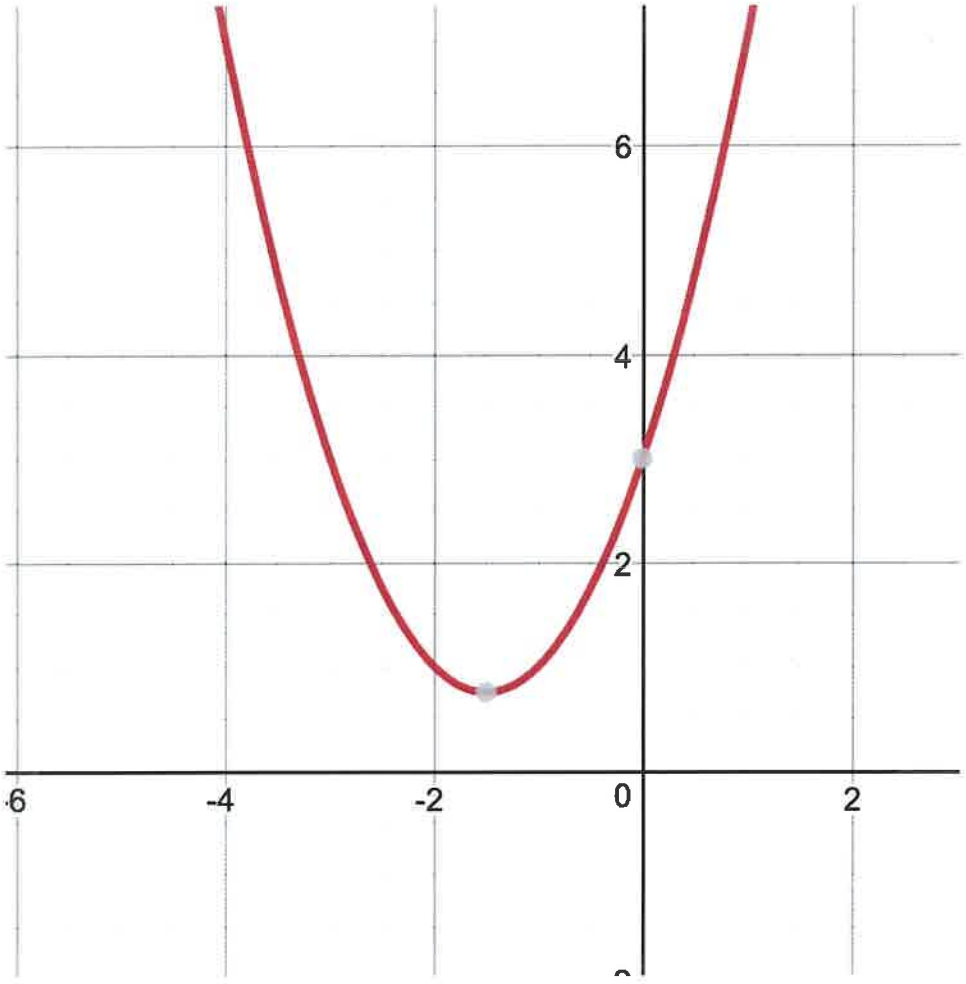
x-intercepts

$h(x) = (-2x+3)(x+2)$, $(\frac{3}{2}, 0)$, $(-2, 0)$

Graph the function $g(x) = x^2 + 3x + 3$

$= \left(x^2 + 3x + \frac{9}{4}\right) + 3 - \frac{9}{4}$

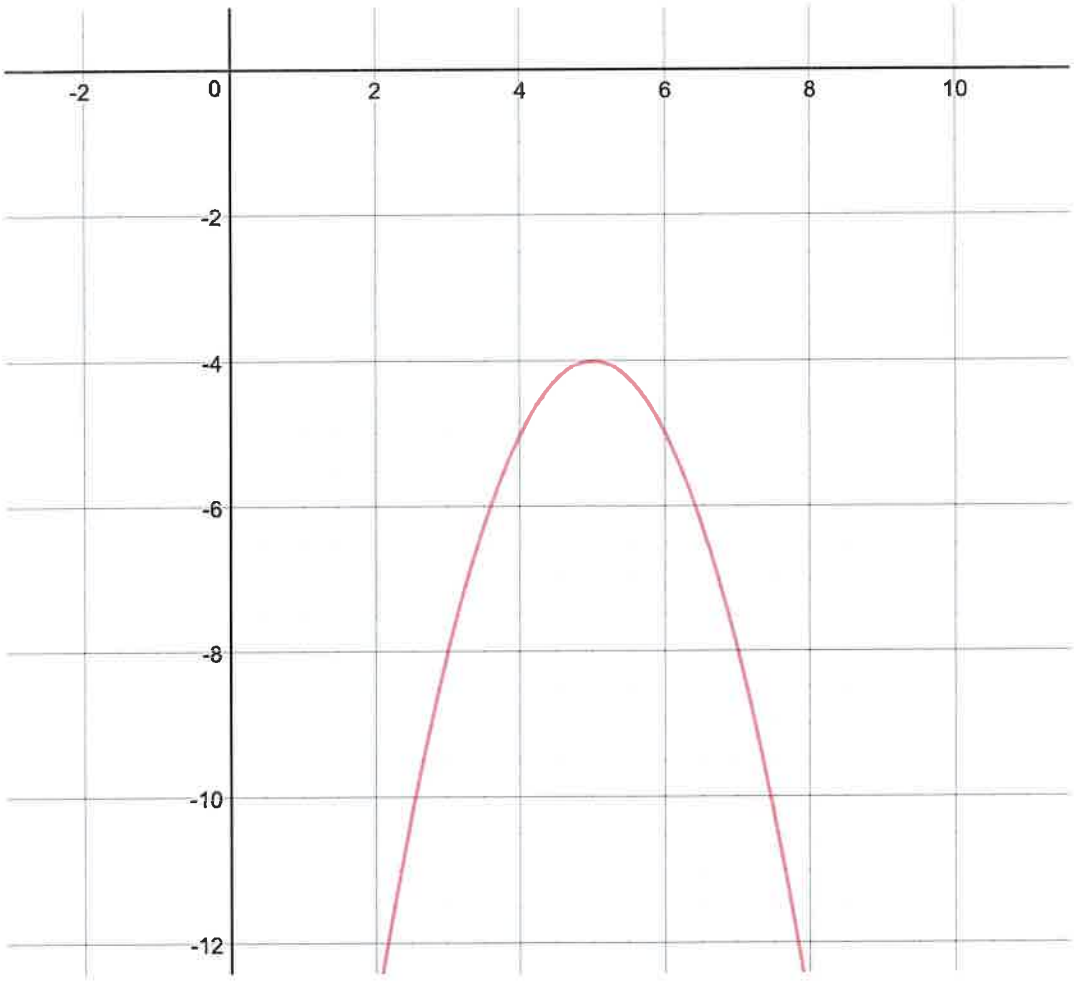
$= \left(x + \frac{3}{2}\right)^2 + \frac{3}{4}$, vertex is $\left(-\frac{3}{2}, \frac{3}{4}\right)$



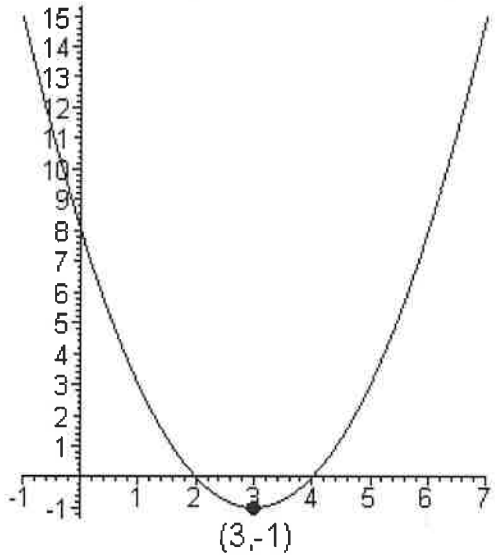
(0,3) is the y-intercept

Standard form of a quadratic function:
The quadratic function $f(x) = a(x-h)^2 + k$
has a graph which is a parabola with a vertex at the point (h, k)
and opens up if $a > 0$ and opens down if $a < 0$.

Graph the function $f(x) = -(x-5)^2 - 4$



Find the equations of the parabolas that are shown below:



vertex is $(3,-1)$

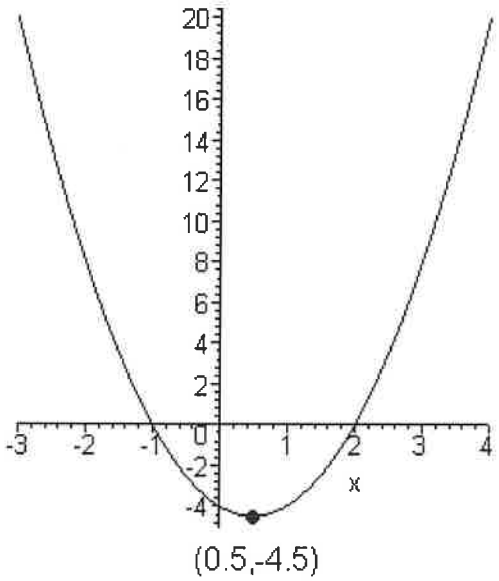
y-intercept is $(0,8)$

$y = a(x-3)^2 - 1$

$8 = a(-3)^2 - 1$

$9 = 9a,\ a = 1$

$y = (x-3)^2 - 1$



vertex is $\left(\frac{1}{2}, -\frac{9}{2}\right)$

x intercepts are $(-1,0)$, $(2,0)$

$y = a(x+1)(x-2)$

when $x = \frac{1}{2}$, $y = -\frac{9}{2}$

$-\frac{9}{2} = a\left(\frac{3}{2}\right)\left(-\frac{3}{2}\right),\ -18 = -9a$

$a = 2$

$y = 2(x+1)(x-2) = 2x^2 - 2x - 4$

Max and Min Problems:

The height of a ball (in meters) that is tossed up into the air from a starting height of 1.8 meters with an initial velocity of 24.5 meters per second is given by the function $s(t) = 1.8 + 24.5\,t - 4.9\,t^2$.

What is the maximum height that is obtained by the ball?

we must find the vertex.

$$s(t) = -4.9\left[t^2 - 5t\right] + 1.8$$

$$= -4.9\left[t^2 - 5t + \frac{25}{4}\right] + 1.8 + 4.9\left(\frac{25}{4}\right)$$

$$= -4.9\left[t - \frac{5}{2}\right]^2 + 63.05$$

Maximum height is 63.05 meters.