

Nonlinear Systems of Equations

Identify the solutions to the system of equations and then verify the solutions algebraically.

$$(x - 5)^2 + (y - 2)^2 = 9$$

$$(x - 3)^2 + (y - 2)^2 = 25$$

$$(x - 5)^2 - (x - 3)^2 = -16$$

$$x^2 - 10x + 25 - (x^2 - 6x + 9) = -16$$

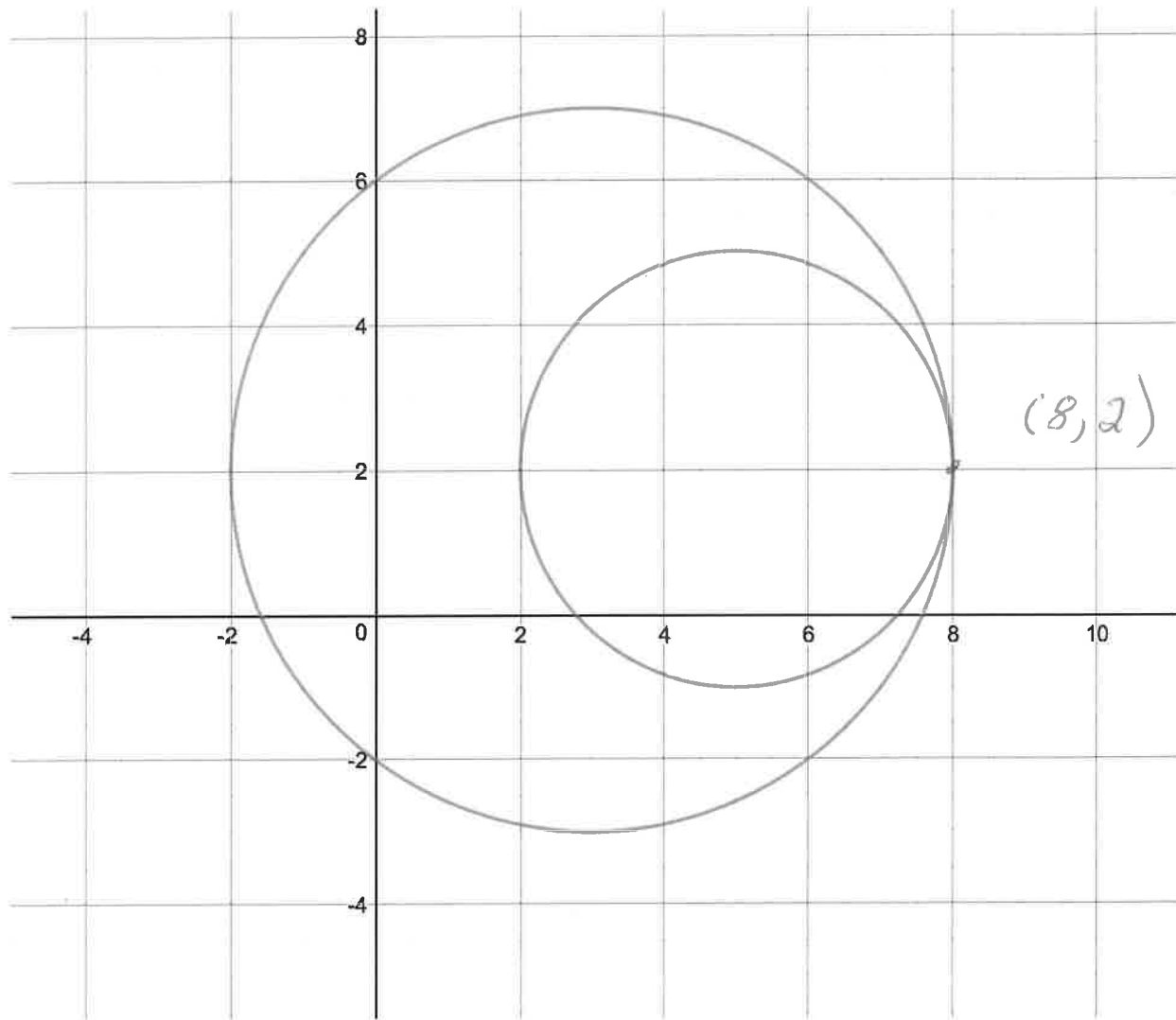
$$-4x + 16 = -16, \quad -4x = -32$$

$$x = 8, \quad 9 + (y - 2)^2 = 9$$

$$(y - 2)^2 = 0, \quad y = 2$$

$x = 8, y = 2$ is the solution.

Note: The point $(8, 2)$ is on both circles.



Solve the system of equations:

$$x^2 + y^2 = 2$$

$$y = -x^2$$

$$x^2 + (-x^2)^2 = 2$$

$$x^4 + x^2 - 2 = 0$$

$$\text{Let } w = x^2, \quad w^2 + w - 2 = 0$$

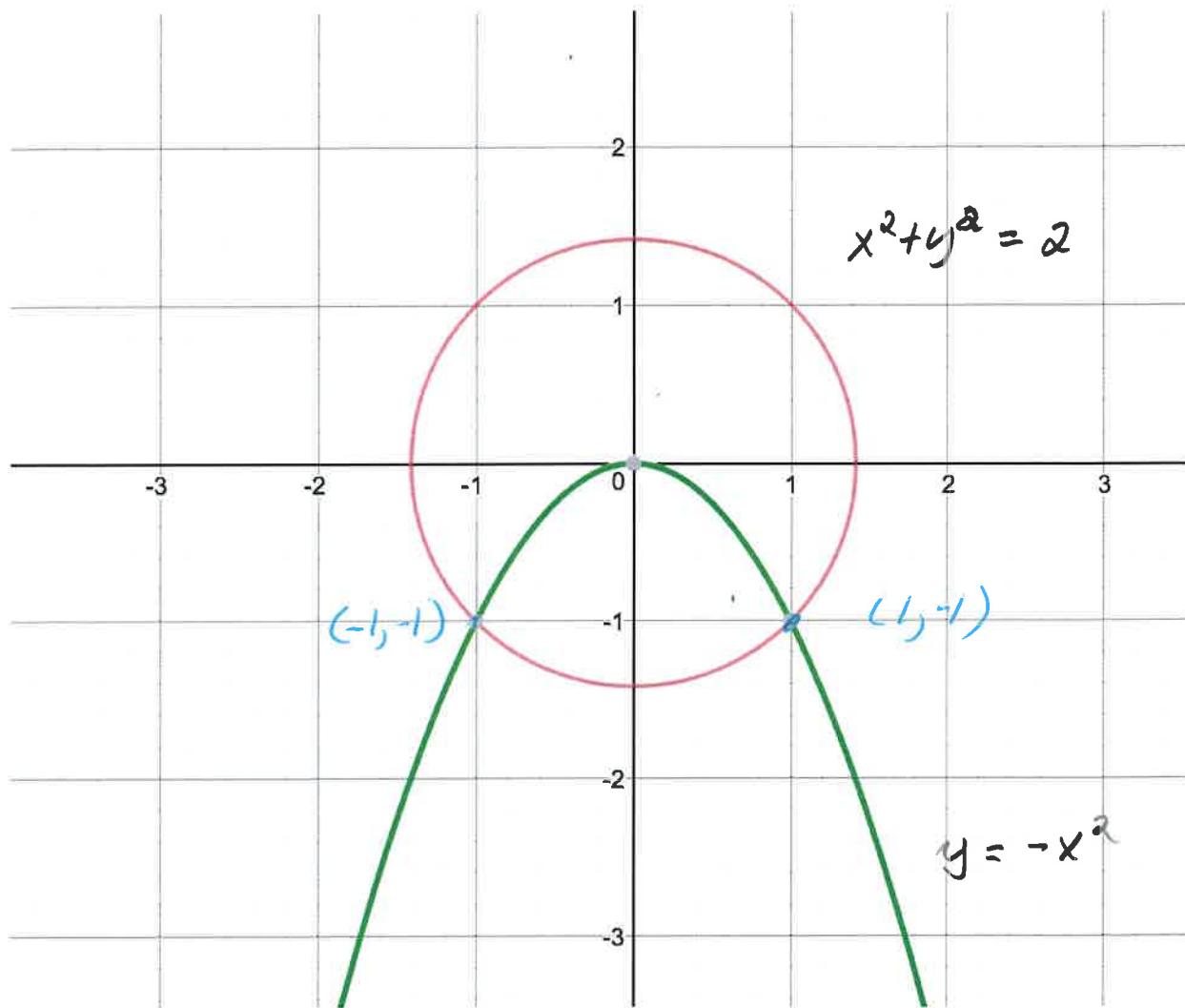
$$(w+2)(w-1) = 0, \quad w = -2 \text{ or } w = 1$$

$$x^2 = -2 \quad \text{or} \quad x^2 = 1$$

$$x = 1 \text{ or } x = -1, \quad y = -x^2 \quad \text{so}$$

$(1, -1)$ is a solution

$(-1, -1)$ is also a solution.



Identify the solutions to the system of equations and then verify the solutions algebraically.

$$y + 2 = \frac{1}{2}(x - 1)^2$$

$$y + 4 = (x - 1)^2$$

$$2y + 4 = (x - 1)^2$$

$$y + 4 = (x - 1)^2$$

$$y = 0$$

$$(x - 1)^2 = 4$$

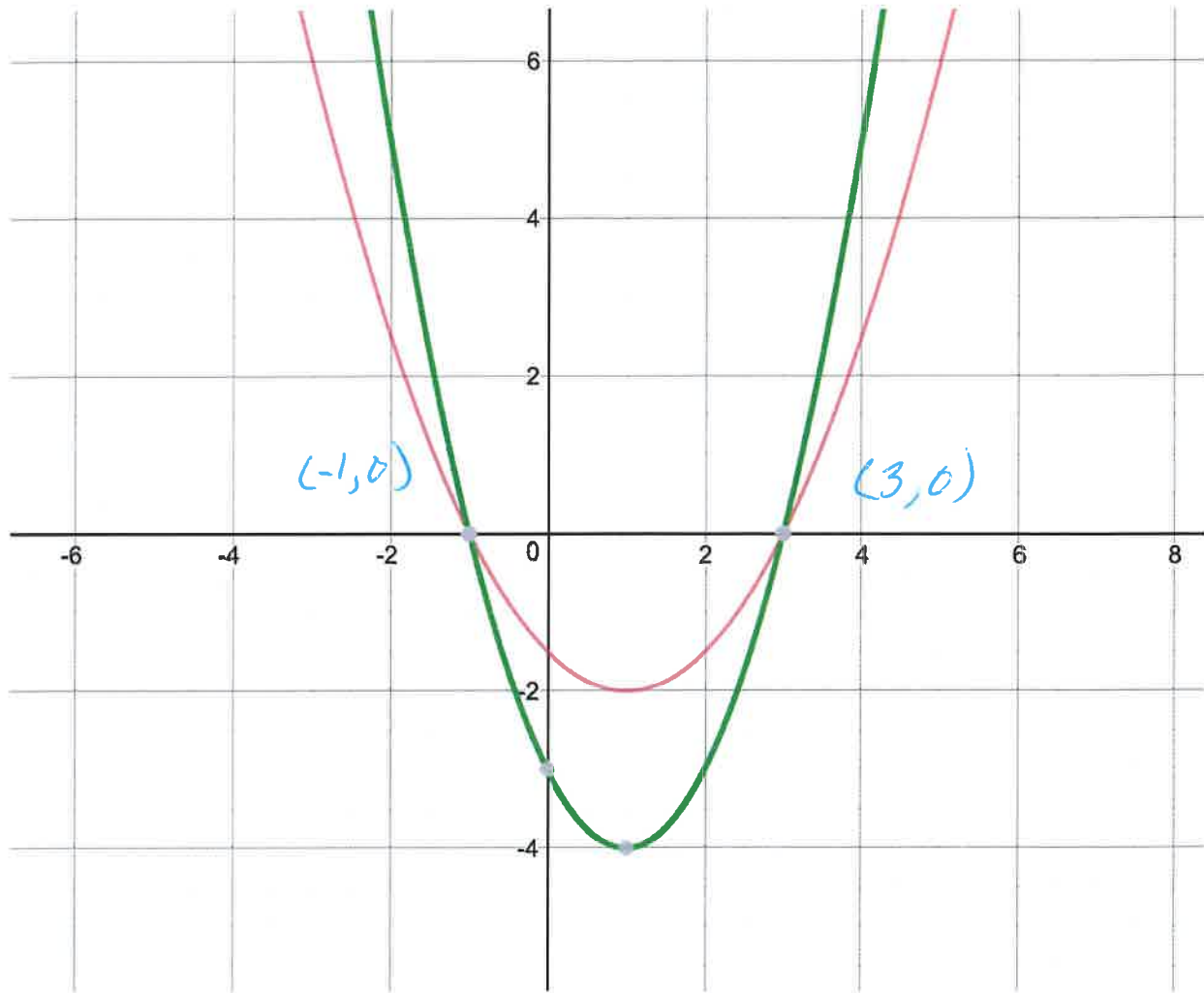
$$x - 1 = 2 \text{ or } x - 1 = -2$$

$$x = 3 \text{ or } x = -1$$

$x = 3, y = 0$ is a solution

$x = -1, y = 0$ is a solution

Note: the points $(3, 0)$ and $(-1, 0)$ are both on the two parabolas.



Solve the system of equations:

$$2x^2 + y^2 = 11$$

$$2(x+2)^2 + y^2 = 27$$

$$2x^2 + y^2 - 2(x+2)^2 - y^2 = -16$$

$$2x^2 - 2(x+2)^2 = -16$$

$$x^2 - (x+2)^2 = -8$$

$$x^2 - (x^2 + 4x + 4) = -8$$

$$-4x - 4 = -8, \quad -4x = -4$$

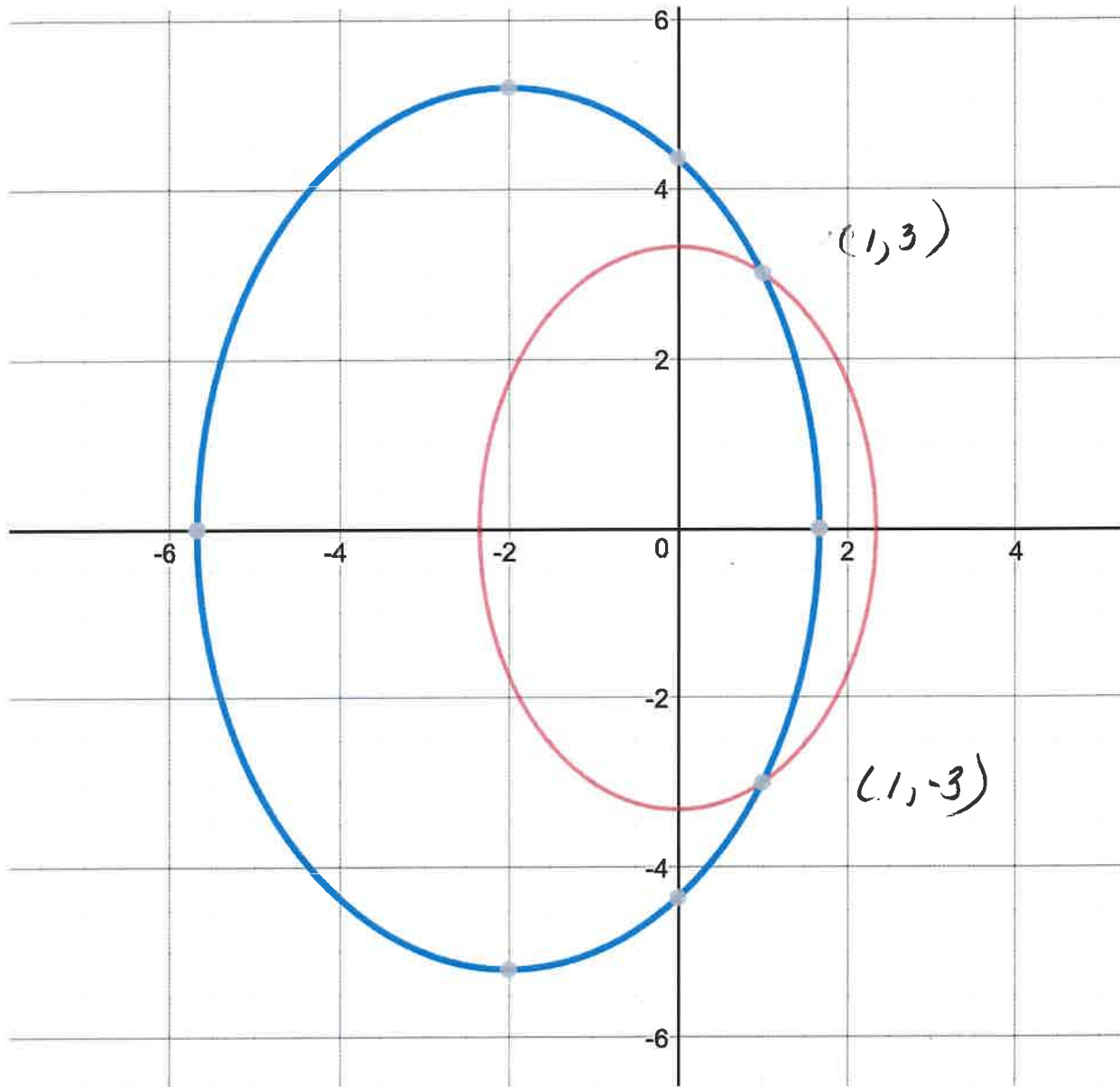
$$x = 1 \quad 2(1)^2 + y^2 = 11$$

$$2 + y^2 = 11, \quad y^2 = 9, \quad y = \pm 3$$

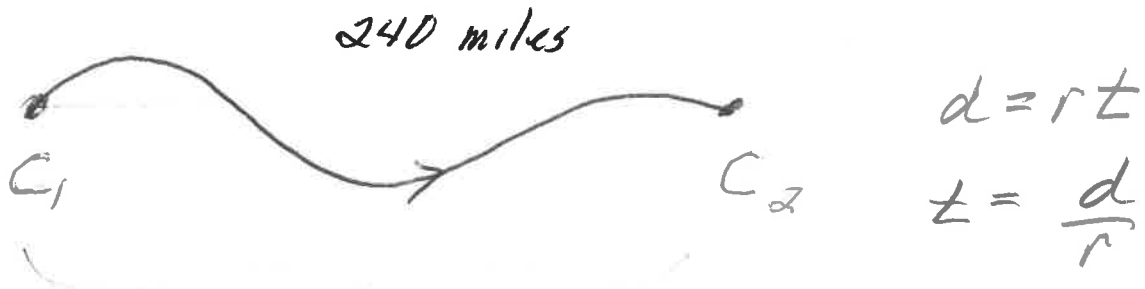
$x = 1, y = 3$ is a solution

$x = 1, y = -3$ is a solution.

$(1, 3)$ and $(1, -3)$ are points
on both ellipses.



James takes 20 minutes longer than Sue to make the 240-mile drive between two cities. Sue drives three mph faster. How fast do James and Sue drive?



Let x = speed (rate) of James

y = speed (rate) of Sue

$$y = x + 3 \quad \frac{240}{x} = \frac{240}{y} + \frac{1}{3} \quad \underline{\underline{20 \text{ min}}}$$

$$y = x + 3, \quad \frac{240}{x} = \frac{240}{x+3} + \frac{1}{3}$$

$$240(x+3) = 240x + \frac{x(x+3)}{3}$$

$$240x + 720 = 240x + \frac{x^2 + 3x}{3}$$

$$x^2 + 3x - 2160 = 0$$

$$x = \frac{-3 \pm \sqrt{9 + 4(2160)}}{2}$$

$$x = 45 \text{ mph}, \quad y = 48 \text{ mph}$$

$$5\frac{1}{3} \text{ hours}$$

$$5 \text{ hours}$$