Section 1.3 Polynomials and Factoring

A **polynomial** is an expression that can be written in the form:

$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + ... + a_1 x + a_0$

Where n is a whole number. The degree of a polynomial in this form is n.

Examples:

Subtract: $(2x^3 - 3x^2 - 4) - (2 - 3x + x^2 + x^3)$

Multiply: $(x^2 - 2x + 2)(x^2 - 3)$

$3x^2 + 2x$

$16x^3 + 6x^2 - 12$

$4x^3y - 2xy^3$

$12(x - 2) + 4x(x - 2)$

Sometimes a polynomial has a greatest common factor of 1 but can still be factored by using other processes. One of these processes is called **factoring by grouping**. Factoring by grouping is a process that should be considered when trying to factor a polynomial of four or more terms.

Example:
Factor the polynomial

$$2x^3 + 4x^2 + 3x + 6$$

$$12x^2y + 24x^2z + 2xy + 4xz$$

The main thing to remember when trying to **factor a trinomial** into the product of two binomials is that the leading term of the trinomial must be the product of the leading terms of the two binomials and that the last term of the trinomial must be the product of the last terms of the two binomials.

Factor the trinomials below into a product of two binomials using the information above:

$x^2 + 2xy - 24y^2$

$6x^2 - 13x + 6$

Grouping Method for Factoring a Polynomial of the form $ax^2 + bx + c$:

1. Find a pair of numbers m,n whose product is ac and whose sum is b.

2. Write the polynomial in the form $ax^2 + mx + nx + c$ and use the grouping method to factor.

Factor the polynomials below using the AC Method:

$$4x^2 - 12x + 5$$

$$-12a^4 + 52a^2 + 9$$

Factoring difference of squares binomials:
$x^2 - y^2 = (x + y)(x - y)$

Factor the following polynomials:

$$9x^2 - 16y^2$$

$$12x^5y^2 - 3x$$

$$81 - (a - 2)^2$$

Factoring sum of cubes and difference of cubes binomials:

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Factor the following polynomials:

$$27a^3 + 1$$

$$16t^4r^3 - 250t$$